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RESEARCH ON NOISE IN CROSSED FIELD DEVICES

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Final Report

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ABSTRACT

The work of this year consisted to study the anomalous effects observed in crossed field guns; the more stricking effect is the excess noise which may reach 10^4 time the thermal noise of the cathode and which causes the sole current in optical systems. The conditions in which such a noise appear are now well known; the spectrum and the correlation in the magnetic field direction have been determined.

The theory of the excess noise has not yet been given; the experiments done show that the noise in the gun proceeds from an instability of the space charge which may be modulated by the classical noise; according to this idea, the first step is to find an unstable flow. Up to now only a flow neglecting the space charge or a flow with constant current density has been studied. In this report a flow in which the cathode current is a linear function of the distance on the cathode has been theoretically studied; the d.c results, obtained with a digital computer, are given in this report; some critical magnetic fields for which the electron velocities vanish after a particular angle constitute an unexpected phenomenon. On the other hand the computed trajectories permit to design a highly convergent gun which could present some advantages compared to the classical gun assuming a constant current density.

The sensitivity of the gun to an external signal has been shown to be very high when the excess noise is present; the noise characteristics of a coaxial optical system have been studied.

It is possible to design a gun without excess noise, by the two following means :

1. A narrow cathode gun for high impedance beams.

2. A gridded gun for low impedance beams (medium power).

Such guns will be used in the T.P.O.M. which will be designed during the second year program to study the parasitic effects in crossed field devices and to realise low noise tubes.

We shall give at first in this final report some characteristic results described in the previous reports

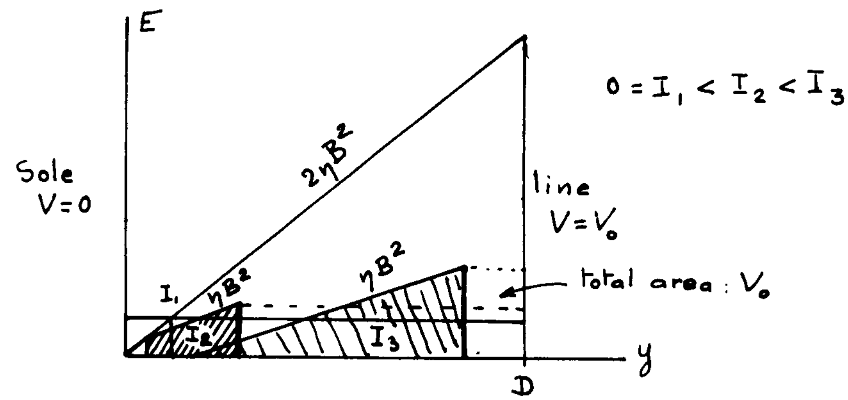
-:-:-

DETAILED REPORT

BRILLOUIN FLOW IN CROSSED FIELD SPACE.

The simplest way to present the properties of such a flow is to plot the electric field versus the distance.

If the sole bias is neglected we have the following sketch, since the slope of the field in the beam is $\frac{\partial E}{\partial y} = \eta B^2$ after the Poisson's law and $\omega_p = \omega_c$, ω_p being the plasma angular frequency and ω_c the cyclotron angular frequency.



When the current injected increases the left hand side of the beam reachs the sole; this corresponds to a first maximum current given by

$$Z_{min} = \left[B/B_c - \left(\left(\frac{B}{B_c} \right)^2 - 1 \right)^{1/2} \right]^{-2}$$

with

$$Z_{min} = \frac{V}{I_{max}} \frac{\eta B \ell / c}{\sqrt{\mu_0 / \varepsilon_0}}$$

ℓ being the width of the structure.

The current can increase again if the left hand side of the beam leaves the sole up to

$$Z_{min} = 1$$

The comparison is done with the theory in the figure 1. When the magnetic field is increased the plate voltage may be raised with a negligeable interception and the cathode current follows the law

$$Z_{min} = 1 \qquad \text{or} \quad I \approx B$$

But the transmission collector over cathode current decreases because the beam spreads(due to the noise.)

MEASUREMENT OF THE NOISE.

a) The sole current, when the sole is negatively biased with respect to the cathode is connected to the noise in the beam and has been used at first to measure the noise; however it may be noticed that, for an infinitely long drift space it is approximately given, from energetic considerations by

$$I_s / I_k = \left[1 - \frac{1}{4}\left(\frac{B_c}{B}\right)^2\right]\left[1 + V_s/V\right]^{-1}$$

Therefore the sole current is a measurement of the noise only for short sole length,or with a segmented sole which indicates at what distance from the gun the beam reach the sole.

b) Use of a cavity or a delay line.

The microwave noise may be collected by a forward wave structure (undriven TPOM) or by a backward wave circuit (M Carcinotron below the starting current); in the second case the noise appears like a narrow band noise ($\Delta f/f$ = 5 to 10 %) the central frequency of which being the oscillation frequency of the carcinotron. However the position of the beam is not known with a sufficient accuracy nor the coupling between the beam and the circuit; a cavity would present the same difficulty.

c) Measurement of the low frequency noise.

Non linear effects in the beam involves a low frequency noise which is strickly connected to the microwave noise, and the noisiness of the beam has therefore been studied by measuring the noise of the collector current.

Two different methods have been utilized :

1. The collector being grounded through a 50 Ω resistance the noise voltage across it is measured between 30 KHz and 230 MHz with a tunable comercial available amplifier having a bandwidth of 4 KHz (Bruel and Kjoer Nº 2002 and 2004). For the noiseness of the beam we shall use the noise modulation N which is the ratio of the r.m.s.

noise current in a 4 Kc band to the d.c. current. The total noise modulation integrated over the total band can reach unity e.g. the collector current is completely modulated by noise. Most of the measurements have been done with this method.

2. The second method utilised a low frequency passive tunable resonance circuit in the collector and the noise is measured on an oscillograph. This method avoids non linear effects in the amplifier and is therefore more accurate but less sensitive.

NOISE VERSUS THE TEMPERATURE OF THE CATHODE.

It is well known that the temperature of the cathode has a strong effect on the noise; this is shown again in the Fig.2 with an impregnated cathode, the collector current being kept constant. The excess noise vanishes under 1000°, which corresponds to the temperature limitation of the cathode when no magnetic field is applied.

THE NOISE NEAR THE GUN HAS BEEN PHOTOGRAPHIED WITH A WIDE BAND AND HIGH SPEED OSCILLOSCOPE.

The oscillograms are reproduced approximately in the Fig.3 which shows that the noise appears like an oscillation modulated by noise, when B/B_c is rather small which means when the cathode is not back bombarded. For high magnetic fields the fundamental frequency disappears.

Such a high frequency oscillation ($f \simeq 180$ MHz with $B = 30$ Gauss) seems to be modulated by an other frequency ($F \simeq 40$ MHz) which may be due to a feedback from the collector région.

NOISE AFTER A LONG DRIFT SPACE.

After a long drift space the noise is more similar to a white noise; the theory of the diocotron effect shows that the gain is

$$\gamma_{Neper/m} = \frac{\omega}{v} \frac{1}{Z} \left(\frac{B}{B_c}\right)^2$$

It is proportionnal to ω; however, experiments done on the break up of hollow beams by B. EPSZTEIN[1] and others, show the large signal behavior of the beam; it forms a set of spokes which rotates one around the other; the spokes are of increasing size the limit being due to the image effect in the sole and the line. This large signal effects involve low frequency components; the spokes being distributed at random, let us consider each of them as a ponctual charge q the rms current will be

$$\sqrt{\overline{i^2}} = 2 q I \Delta f$$

(1) B. EPSZTEIN : Thèse à l'Université de Paris.

With such an asumption the spectrum will be flat from 0 up to a frequency corresponding to the mean distance of the spokes. The measurements of the noise spectrum indicate that the size should be of the order of magnitude of the line sole distance D. This characteristic frequency lies between v/2D and v/D as shown in the Fig.4.

SCALING LAWS.

From the previous paragraph it may be expected that the total noise modulation $N_T = \sqrt{\overline{i_T^2}} / \overline{i}$ is constant and the bandwidth increases according to B when a scaling in voltage is done with $B \sim V^{1/2}$; consequently the measured noise modulation in a fixed bandwidth (4 KHz) should decreases according to $N \sim V^{-1/4}$. The experimental curve 5 shows that the mean value decrease slowly with V but the range of V is not sufficient to establishe that it decreases according to $V^{-1/4}$. Some ripples are observed which could be due to a feedback from the collector. At the lowest voltages, the sole current increases rapidly, but N decreases; we may suppose that the electrons which have gained or lost energy are absorbed so that the other electrons are less noisy.

STUDY OF THE CORRELATION.

We intended to know if the noise is correlated in the direction of the magnetic field. We may expect that the correlation is unity for distances smaller than the height of a cycloid and zero for much larger distances. The experiments done with collectors near the gun are not in disagreement with this hypothesis; after a long drift

space, the correlation increases as it may be expected from the theory of the diocotron gain.

The correlation is measured at low frequencies; its value does not depend on the particular frequency chosen. A special apparatus has been built for this purpose.

An exemple of the results is shown in the figure 6, for from the gun (b) the correlation is unity for small B (the correlation may be due to the transverse diocotron gain) but it tends to vanish at the highest B/B_c (small height of the cycloïd). Near the gun, the distance between the collectors is rather large so that the correlation is always zero.

Some tests have been done at first with probes on the cathode; the tests have not been successfull for, on one hand, at the high frequencies cold parasitic couplings exist between the probes and, on the other hand the low frequency components have a too small amplitude in the gun.

SPECTRUM WITH AN r.f. SHORT CIRCUIT BETWEEN THE CATHODE AND THE SOLE.

The spectrum generally observed is shown in the figure 7 (curve without capacities); it presents periodic peaks (the separation into two peaks of the three first peaks has no meaning; it is due to the image frequency).

When an r.f. short circuit is put between the plate and the cathode (the decoupling is indicated in the upper curve). This support the hypothesis that the periodicity is due to the collector r.f. current flowing back to the plate and inducing a voltage which modulates in phase or in opposite phase the beam according to the frequency. This is observed for moderate magnetic fields; for high B/B_c, the phase of the noise along the cathode is probably to much erratic so that the plate r.f. voltage (obviously in phase in all the gun since the sizes are small compared to the free wavelength) does not involve a clear periodicity.

TRAJECTORIES AND NOISE WITH A SCREEN GRID.

The figure 8 shows a gridded gun; the grid is constituted of thin tantalum tapes through which the beam flows. The cycloïdal movement may be suppressed with a suitable voltage, even with a magnetron type gun.

The figure 9 shows a gun gridded with a thin nickel grid; the technological results are not good.

The figure 10 shows the optical system used with a tantalum grid, and with some cathode probes.

The figure 11 shows the noise modulation and the current with such a grid. In contrast with the classical gun, the excess noise

appears only when $B/B_c = 1{,}6$; one remarks that the sole current appears just at the same time, the collector current decreases for more current is absorbed by the grid and the collector current increases for higher magnetic fields when the beam flows between the cathode and the grid.

The figure 12 gives a summary of the results with gridded guns.

TEST WITH A COAXIAL TYPE STRUCTURE.

The photograph of the experimental system is shown in the figure 13.

An experiment (figure 14) shows the noise modulation which appears like the sole current only when $B/B_c > 1{,}4$. A 100 % transmission is obtained from $B/B_c = 1{,}1$ to 1.4.

A set of anodes permits to measure the noise variation from the gun; with a small initial noise the diocotron effect is clearly seen; but the gun may be saturated by the noise; the limit value of the noise N is in any cases around $0{,}5.10^{-3}$.

SENSITIVITY OF THE GUN TO AN r.f. EXTERNAL SIGNAL.

We found that the beam modulation is 30 dB higher when the excess noise is present, the frequency of the signal being around 30 to 50 MHz; this external signal was applied by a wire on the cathode parallel to the magnetic field. The beam modulation was observed on the collector.

CYCLOTRON RESONANCES.

Cyclotron resonances are observed when the beam flows outside the optical system in low d.c. field regions; the residual gas may in this case play a role.

DESIGN OF A GRIDDED GUN.

The power dissipated on the grid was in the previous experiments 1 % of the total beam power; in order to decrease it, the grid must be nearer to the cathode which corresponds to a smaller grid-cathode potential. However a constant d.c. field is measured on the cathode only if the wire spacing is smaller than the grid to cathode distance.

So, we have built a grid using a pitch of 70 μ and a wire diameter of 10 μ. It was at first at 0,5 mm of an impregnated cathode (22 x 3 mm) Plate-cathode spacing = 3.9 mm.

The secondary emission and the direct emission of the grid involves a grid current which decreases with increasing cathode currents.

Without magnetic field one has

V_{p_v}	V_{g_v}	I_p mA	I_g mA	grid power/Beam power
200	0	1.7	0.15	
	10	23	3	0.6 %
	20	61	6	
375	24	99	3.4	

RESULTS IN OPTICAL SYSTEMS.

The line is connected to the accelerating plate.

The figure shows the maximum grid voltage which may be applied without sole current; for lower voltages a small grid current may appears also, so that the voltage V_g indicated is the best one; by no sole current we mean less that 1 % of the cathode current.

We see that a noiseless beam power of 525^V x 110 mA may be applied with a good transmission (the line and collector current are measured together but we may suppose that the line current is negligeable since the sole current is zero).

The figure 17 shows the results with 130 gauss. The too scales are modified to take into account the scaling laws. The corresponding voltages are much higher so that an important part of the beam reachs the accelerating plate.

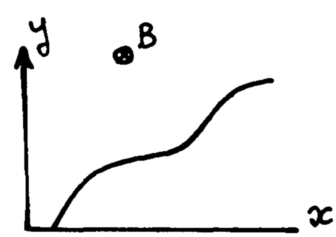
TRAJECTORIES IN A CROSSED FIELD GUN WITH A CURRENT DENSITY PROPORTIONNAL TO THE RADIUS

The calculations of the noise which assumes the uniformity in the direction of the cathode surface have not led up to now to instabilities or to high diocotron gains. This is the reason for the study of a beam in which the current varies linearly with the distance because this is more similar to the real situation of the M type gun when the excess noise appear; such a slipping stream may involve high diocotron gain mainly near the pole.

In the case of a uniform current density the trajectories are given (the initial velocities being neglected) by :

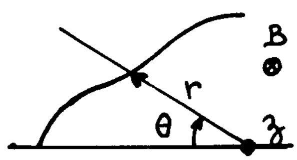
$$x - x_0 = \frac{\eta J}{\varepsilon \omega_c^3}\left[\frac{(\omega_c t)^2}{2} + \cos \omega_c t - 1\right]$$

$$y = \frac{\eta J}{\varepsilon \omega_c^3}\left[\omega_c t - \sin \omega_c t\right]$$



Let us consider the case where the current varies according to r ; it may be easily shown that all the trajectories are homothetical with respect to the pole, that the space charge density ρ is independant on r and that the potential varies according to r^2, as long as there is no multiple stream (this occurs when V_0 vanishes).

B is perpendicular to the paper and uniform; the potential vector A is supposed to have only a component equal to



$$A_\theta = \frac{r}{2} B$$

which satisfies $\vec{B}$ = grad $\vec{A}$

In this kind of problem it is known(1) that the flow is irrotational, $\mathrm{rot}\,(mv - eA) = 0$, so that $mv - eA$ is the gradient of a function W.

$$v = \mathrm{grad}\, W + \eta A$$

or

$$V_r = \frac{\partial W}{\partial r}$$

$$V_\theta = \frac{1}{r}\frac{\partial W}{\partial \theta} + \omega r$$

with $\omega = \frac{\eta B}{2}$ (Larmor angular frequency)

$$V_z = 0$$

Let us put

$$W = \omega r^2 g(\theta)$$

(1) GABOR, P.I.R.E. Vol 33 (1945) p. 792.

for the asymptotic developpement near the cathode it will be easier to put $f(\theta)-\theta$ instead of $g(\theta)$; the velocity is now given by :

$$W = \omega r^2 \left(f(\theta)-\theta\right)$$
$$V_r = 2\omega r \left(f(\theta)-\theta\right)$$
$$V_\theta = \omega r\, f'(\theta)$$
$$V_z = 0$$

The initial condition at the cathode (zero velocity) involves

$$f(0) = f'(0) = 0$$

The problem is now to find $f(\theta)$

The potential is then given by

$$2\eta\Phi = \omega^2 r^2\left[4f^2 - 8f\theta + 4\theta^2 + f'^2\right]$$
$$\eta\frac{\partial\phi}{\partial r} = \omega^2 r\left[4f^2 - 8f\theta + 4\theta^2 + f'^2\right]$$
$$\eta\frac{\partial^2\phi}{\partial r^2} = \omega^2\left[4f^2 - 8f\theta + 4\theta^2 + f'^2\right]$$
$$\frac{2}{r}\eta\frac{\partial\phi}{\partial\theta} = \omega^2 r\left[8ff' - 8f'\theta - 8f + 8\theta + 2f'f''\right]$$
$$\frac{1}{r^2}\eta\frac{\partial^2\phi}{\partial\theta^2} = \omega^2\left[4f'^2 + 4ff'' - 4f''\theta - 4f' - 4f' + 4 + f''^2 + f'f'''\right]$$
$$\eta\,\Delta\Phi = -\frac{\eta}{\varepsilon_0}\rho = \omega^2\left[8f^2 - 16f\theta + 8\theta^2 + 6f'^2 + 4 + 4ff'' - 4f''\theta - 8f' + f''^2 + f'f'''\right]$$

The components of the current density j_r and j_θ are :

$$
-\frac{\eta}{\varepsilon_0} j_r = -\frac{\eta}{\varepsilon_0}\rho V_r = 2\omega^3 r \begin{bmatrix} 8f^3 - 16f^2\theta + 8f\theta^2 + 6f'^2 f \\ + 4f^2 f'' - 4ff''\theta - 8ff' + 4f + ff''^2 \\ + ff'f''' - 8f^2\theta + 16f\theta^2 - 8\theta^3 - 6f'^2\theta \\ - 4ff''\theta + 4f''\theta^2 + 8f'\theta - 4\theta \\ - f''^2\theta - f'f'''\theta \end{bmatrix}
$$

$$
-\frac{\eta}{\varepsilon_0} j_r = 2\omega^3 r \begin{bmatrix} 8f^3 + 4f^2 f'' + 6ff'^2 + ff''^2 + ff'f''' \\ -24f^2\theta - 6f'^2\theta - 8ff''\theta - f''^2\theta - f'f'''\theta - 8ff' \\ + 24f\theta^2 + 4f'\theta^2 + 8f'\theta + 4f - 4\theta - 8\theta^3 \end{bmatrix}
$$

$$
-\frac{\eta}{\varepsilon_0} j_\theta = -\frac{\eta}{\varepsilon_0}\rho V_\theta = \omega^3 r \begin{bmatrix} 8f^2 f' - 16ff'\theta + 8f'\theta^2 + 6f'^3 \\ + 4ff'f'' - 4f'f''\theta - 8f'^2 + 4f' \\ + f'f''^2 + f'^2 f''' \end{bmatrix}
$$

Let us write the law :

$$
\operatorname{div} \bar{j} = \frac{1}{r}\frac{\partial}{\partial r}(r j_r) + \frac{1}{r}\frac{\partial j_\theta}{\partial \theta} = 0
$$

The two terms are :

$$-\frac{\eta}{\varepsilon_0}\frac{1}{r}\frac{\partial}{\partial r}\left(r\, j_r\right) = 4\omega^3 \Big[8 f^3 + 4 f^2 f'' + 6 f f'^2 + f f''^2 + f f' f''' - 24 f^2 \theta - 6 f'^2 \theta - 8 f f'' \theta - f''^2 \theta - f' f''' \theta - 8 f f' + 24 f \theta^2 + 4 f'' \theta^2 + 8 f' \theta + 4 f - 4\theta - 8\theta^3 \Big]$$

$$-\frac{\eta}{\varepsilon_0}\frac{1}{r}\frac{\partial}{\partial \theta}\left(j_\theta\right) = \omega^3 \Big[16 f f'^2 + 8 f^2 f'' - 16 f'^2 \theta - 16 f f'' \theta - 16 f f' + 8 f' \theta^2 + 16 f' \theta + 18 f'^2 f'' + 4 f'^2 f'' + 4 f f''^2 + 4 f f' f''' - 4 f''^2 \theta - 4 f' f''' \theta - 4 f f''' - 16 f' f'' + 4 f'' + 4 f' f'' f''' + f''^3 + f'^2 f^{IV} \Big]$$

$$= \omega^3 \Big[16 f f'^2 + 8 f^2 f'' + 22 f'^2 f'' + 4 f f' f''' + 4 f f''^2 + 4 f' f'' f''' + f''^3 + f'^2 f^{IV} - 16 f f' - 20 f' f'' + 4 f'' - 16 f'^2 \theta - 16 f f'' \theta - 4 f''^2 \theta - 4 f' f''' \theta + 16 f' \theta + 8 f'' \theta^2 \Big]$$

This led to the differential equation in $f(\theta)$

$$\left.\begin{aligned} &32 f^3 + 24 f^2 f'' + 40 f f'^2 + 8 f f''^2 + 8 f f' f''' + 22 f'^2 f'' \\ &+ 4 f' f'' f''' + f''^3 + f'^2 f^{IV} - 48 f f' + 16 f - 20 f' f'' + 4 f'' \\ &+ \theta\left[-96 f^2 - 40 f'^2 - 48 f f'' - 8 f''^2 - 8 f' f''' + 48 f' - 16\right] \\ &+ \theta^2\left[96 f^2 + 24 f''\right] - 32\theta^3 \end{aligned}\right\} = 0$$

Let us develop $f(\theta)$ in sery for small θ, taking into account the initial conditions

$$V_r(0) = 0 \qquad V_\theta(\theta = 0) = 0$$

One obtains :

$$f = \gamma_5\,\theta^{5/3} + \gamma_7\,\theta^{7/3} + \gamma_9\,\theta^{9/3} + \gamma_{11}\,\theta^{11/3}$$

that is

$$f' = \frac{5}{3}\gamma_5\,\theta^{2/3} + \frac{7}{3}\gamma_7\,\theta^{4/3} + \frac{9}{3}\gamma_9\,\theta^{6/3} + \frac{11}{3}\gamma_{11}\,\theta^{8/3}$$

$$f'' = \frac{10}{9}\gamma_5\,\theta^{-1/3} + \frac{28}{9}\gamma_7\,\theta^{1/3} + \frac{54}{9}\gamma_9\,\theta^{3/3} + \frac{88}{9}\gamma_{11}\,\theta^{5/3}$$

$$f''' = -\frac{10}{27}\gamma_5\,\theta^{-4/3} + \frac{28}{27}\gamma_7\,\theta^{-2/3} + \frac{162}{27}\gamma_9\,\theta^{0} + \frac{440}{27}\gamma_{11}\,\theta^{2/3}$$

$$f^{IV} = +\frac{40}{81}\gamma_5\,\theta^{-7/3} - \frac{56}{81}\gamma_7\,\theta^{-5/3} + \frac{880}{81}\gamma_{11}\,\theta^{-1/3}$$

One sees that the derivation of high order reach for $\theta \to 0$ values very much higher than the first derivation and than the function itself; since the digital computer begin by $\theta = 0$, some terms will be at first important and others which will be completely negligeable.

The differential equation is written so that the important terms (for $\theta \to 0$) are before the less important one.

Then we have :

$$
0 = \left\{
\begin{array}{ll}
 & \text{Power of the first term in} \\
+ f'^2 f^{IV} + 4 f' f'' f''' + f''^3 & \theta^{-1} \\
+ 4 f'' & \theta^{-1/3} \\
- 20 f' f'' - 8 f''^2 \theta - 8 f' f''' \theta & \theta^{+1/3} \\
+ 8 f f''^2 + 8 f f' f''' + 22 f'^2 f'' - 16 \theta & \theta^{+1} \\
+ 16 f + 48 f' \theta + 24 f'' \theta^2 & \theta^{+5/3} \\
- 48 f f' - 48 f f'' \theta - 40 f'^2 \theta & \theta^{+7/3} \\
+ 24 f^2 f'' + 40 f f'^2 - 32 \theta^3 & \theta^{+9/3} \\
+ 96 f \theta^2 & \theta^{+11/3} \\
- 96 f^2 \theta & \theta^{+13/3} \\
+ 32 f^3 & \theta^{+15/3}
\end{array}
\right.
$$

Since the second third and fourth derivative of $f(\theta)$ become infinite when θ approaches 0 , we have to begin the calculation, not at $\theta = 0$ but for a small value of θ ; so, we need the first terms of the developpement of $f(\theta)$.

The multiplication of the various derivative gives :

$$
f'^2 f^{IV} = \frac{\theta^{-1}}{729} \left\{ \begin{array}{l} 1000\,\gamma_5^3 + 1400\,\gamma_5^2\gamma_7\,\theta^{2/3} \\ + (3.600\,\gamma_5^2\gamma_9 - 1960\,\gamma_5\gamma_7^2)\,\theta^{4/3} \\ + (26.400\,\gamma_5^2\gamma_{11} - 2744\,\gamma_7^3 + 0 \times \gamma_5\gamma_7\gamma_9)\,\theta^{6/3} + \cdots \end{array} \right.
$$

$$
4 f' f'' f''' = \frac{\theta^{-1}}{729} \left\{ \begin{array}{l} -2000\,\gamma_5^3 - 2.800\,\gamma_5^2\gamma_7\,\theta^{2/3} \\ + (15680\,\gamma_5\gamma_7^2 + 18000\,\gamma_5^2\gamma_9)\,\theta^{4/3} \\ + (21952\,\gamma_7^3 + 151200\,\gamma_5\gamma_7\gamma_9 + 66000\,\gamma_5^2\gamma_{11})\,\theta^{6/3} + \cdots \end{array} \right.
$$

$$
f''^3 = \frac{\theta^{-1}}{729} \left\{ \begin{array}{l} 1000\,\gamma_5^3 + 8400\,\gamma_5^2\gamma_7\,\theta^{2/3} \\ + (16200\,\gamma_5^2\gamma_9 + 23520\,\gamma_5\gamma_7^2)\,\theta^{4/3} \\ + (21952\,\gamma_7^3 + 26400\,\gamma_{11}\gamma_5^2 + 90.720\,\gamma_5\gamma_7\gamma_9)\,\theta^{6/3} \\ + \cdots \end{array} \right.
$$

The first comparison of the coefficients of the terms in θ^{-1} gives

$$1000\gamma_5^3 - 2000\gamma_5^3 + 1000\gamma_5^3 = 0$$

is not determined by the differential equation but only by the initial conditions. For the second comparison ($\theta^{-1/3}$) we need

$$4f'' = \frac{\theta^{-1}}{729}\left[3240\gamma_5\,\theta^{2/3} + 9072\gamma_7\,\theta^{4/3} + 17496\gamma_9\,\theta^{6/3} + \cdots\right]$$

which gives

$$1400\gamma_5^2\gamma_7 - 2800\gamma_5^2\gamma_7 + 8400\gamma_5^2\gamma_7 + 3240\gamma_5 = 0$$

$$\gamma_7 = -\frac{3240}{7000}\cdot\frac{1}{\gamma_5} = -\frac{81}{175}\cdot\frac{1}{\gamma_5}$$

The third comparison (terms in $\theta^{1/3}$) implies the knowledge of the products :

$$-20f'f'' = +\frac{\theta^{-1}}{729}\left[-27000\gamma_5^2\,\theta^{4/3} - 113400\gamma_5\gamma_7\,\theta^{6/3} + \cdots\right]$$

$$-8f''^2\theta = +\frac{\theta^{-1}}{729}\left[-7.200\gamma_5^2\,\theta^{4/3} - 40320\gamma_5\gamma_7\,\theta^{6/3} + \cdots\right]$$

$$-8f'f'''\theta = +\frac{\theta^{-1}}{729}\left[+3.600\gamma_5^2\,\theta^{4/3} - 5.040\gamma_5\gamma_7\,\theta^{6/3} + \cdots\right]$$

Then,

$$0 = \begin{cases} 3600\gamma_5^2\gamma_9 - 1960\gamma_5\gamma_7^2 + 15680\gamma_5\gamma_7^2 + 18000\gamma_5^2\gamma_9 \\ +16200\gamma_5^2\gamma_9 + 23520\gamma_5\gamma_7^2 + 9072\gamma_7 - 27000\gamma_5^2 \\ -7200\gamma_5^2 + 3.600\gamma_5^2 \end{cases}$$

or

$$37800\,\gamma_5^2\gamma_9 + 37240\,\gamma_5\gamma_7^2 + 9072\,\gamma_7 - 30600\,\gamma_5^2 = 0$$

Consequently

$$\gamma_9 = \frac{306}{378} - \frac{133}{135}\times\frac{\gamma_7^2}{\gamma_5} - \frac{9072}{37.800}\times\frac{\gamma_7}{\gamma_5^2}$$

$$= \frac{17}{21} - \frac{133}{135}\times\frac{81^2}{175^2}\times\frac{1}{\gamma_5^3} + \frac{6}{25}\times\frac{81}{175}\times\frac{1}{\gamma_5^3}$$

$$= \frac{17}{21} - \frac{2187}{21875}\times\frac{1}{\gamma_5^3}$$

For the fourth comparison we need the products

$$8ff''^2 = -\frac{\theta^{-1}}{729}\left[7200\,\gamma_5^3\,\theta^{6/3} + \cdots\right]$$

$$8ff'f''' = \frac{\theta^{-1}}{729}\left[-3.600\,\gamma_5^3\,\theta^{6/3} + \cdots\right]$$

$$22f'^2f'' = \frac{\theta^{-1}}{729}\left[49.500\,\gamma_5^3\,\theta^{6/3} + \cdots\right]$$

$$-16\theta = \frac{\theta^{-1}}{729}\left[-11.664\,\theta^{6/3} + \cdots\right]$$

The comparison gives

$$0 = \begin{cases} +26.400\,\gamma_5^2\gamma_{11} - 2.744\,\gamma_7^3 + 66.000\,\gamma_5^2\gamma_{11} + 21.952\,\gamma_7^3 \\ +151.200\,\gamma_5\gamma_7\gamma_9 + 26.400\,\gamma_5^2\gamma_{11} + 21.952\,\gamma_7^3 + 90.720\,\gamma_5\gamma_7\gamma_9 \\ +17.496\,\gamma_9 - (113.400 + 40.320 + 5.040)\,\gamma_5\gamma_7 \\ +(49.500 + 7.200 - 3.600)\,\gamma_5^3 - 11.664 \end{cases}$$

or

$$\left.\begin{array}{l} 118.800\,\gamma_{11}\gamma_5^2 + 41.160\,\gamma_7^3 + 241.920\,\gamma_5\gamma_7\gamma_9 \\ + 17.496\,\gamma_9 + 53.100\,\gamma_5^3 - 158.760\,\gamma_5\gamma_7 - 11664 \end{array}\right\} = 0$$

Therefore

$$\gamma_{11} = \frac{27}{275}\times\frac{1}{\gamma_5^2} - \frac{147}{110}\times\frac{81}{175}\times\frac{1}{\gamma_5^2} - \frac{59}{132}\gamma_5$$

$$- \frac{81}{550}\left(\frac{17}{21} - \frac{2187}{21875}\times\frac{1}{\gamma_5^3}\right)\times\frac{1}{\gamma_5^2}$$

$$+ \frac{112}{55}\times\frac{81}{175}\left(\frac{17}{21} - \frac{21}{21875}\times\frac{1}{\gamma_5^3}\right)\frac{1}{\gamma_5^2} + \frac{343}{990}\times\frac{\overline{81}^3}{\overline{175}^3}\times\frac{1}{\gamma_5^5}$$

$$\gamma_{11} = -\frac{59}{132}\gamma_5 + \frac{108}{875}\times\frac{1}{\gamma_5^2} - \frac{1.358.127}{30.078.125}\times\frac{1}{\gamma_5^5}$$

Control.

To state the validity of the calculation of the coefficients of the function f we shall proceed by another way.

From the definition

$$V_r = r\omega\left[-2\theta^{3/3} + 2\gamma_5\theta^{5/3} + 2\gamma_7\theta^{7/3} + 2\gamma_9\theta^{9/3} + 2\gamma_{11}\theta^{11/3} + \cdots\right]$$

$$V_r^2 = r^2\omega^2\left[4\theta^{6/3} - 8\gamma_5\theta^{8/3} + (4\gamma_5^2 - 8\gamma_7)\theta^{10/3} + \cdots\right]$$

$$V_\theta = \frac{r\omega}{3}\left[5\gamma_5\,\theta^{2/3} + 7\gamma_7\,\theta^{4/3} + 9\gamma_9\,\theta^{6/3} + 11\gamma_{11}\,\theta^{8/3} + \cdots\right]$$

$$V_\theta^2 = \frac{r^2\omega^2}{9}\left[\begin{array}{l} 25\gamma_5^2\,\theta^{4/3} + 70\gamma_5\gamma_7\,\theta^{6/3} + (49\gamma_7^2 + 90\gamma_5\gamma_9)\theta^{8/3} \\ + (126\gamma_7\gamma_9 + 110\gamma_5\gamma_{11})\,\theta^{10/3} + \cdots \end{array}\right]$$

then the potential Φ is given by

$$2\eta\,\Phi = V_r^2 + V_\theta^2 = \frac{r^2\omega^2}{9}\left\{\begin{array}{l} 25\gamma_5^2\,\theta^{4/3} + (70\gamma_5\gamma_7 + 36)\,\theta^{6/3} \\ + (49\gamma_7^2 + 90\gamma_5\gamma_9 - 72\gamma_5)\,\theta^{8/3} \\ + \left(\begin{array}{l} 126\gamma_7\gamma_9 + 110\gamma_5\gamma_{11} \\ + 36\gamma_5^2 - 72\gamma_7 \end{array}\right)\theta^{10/3} \end{array}\right.$$

and for the derivatives

$$\eta\,\frac{\partial\Phi}{\partial r} = \frac{2r\omega^2}{9}\left[25\gamma_5^2\,\theta^{4/3} + \cdots\cdot\right]$$

$$\eta\,\frac{\partial^2\Phi}{\partial r^2} = \frac{2\omega^2}{9}\left[25\gamma_5^2\,\theta^{4/3} + \cdots\right]$$

$$\frac{\eta}{r}\,\frac{\partial\Phi}{\partial\theta} = \frac{r\omega^2}{27}\left\{\begin{array}{l} 50\gamma_5\,\theta^{1/3} + (210\gamma_5\gamma_7 + 108)\,\theta^{3/3} \\ + (196\gamma_7^2 + 360\gamma_5\gamma_9 - 288\gamma_5)\,\theta^{5/3} \\ + (630\gamma_7\gamma_9 + 550\gamma_5\gamma_{11} + 180\gamma_5^2 - 360\gamma_7)\,\theta^{7/3} \\ + \cdots\cdot \end{array}\right.$$

$$\frac{\eta}{r^2}\cdot\frac{\partial^2\phi}{\partial\theta^2}=\frac{\omega^2}{81}\left[\begin{aligned}&50\,\gamma_5^2\,\theta^{-2/3}+\left(630\,\gamma_5\gamma_7+324\right)\theta^{0}\\&+\left(980\,\gamma_7^2+1.800\,\gamma_5\gamma_9-1.440\,\gamma_5\right)\theta^{2/3}\\&+\left(4.410\,\gamma_7\gamma_9+3.850\,\gamma_5\gamma_{11}+1.260\,\gamma_5^2-2.520\,\gamma_7\right)\theta^{4/3}\\&+\cdots\end{aligned}\right.$$

The space charge density $\rho=-\varepsilon_0\Delta\phi$ is then

$$-\frac{\eta}{\varepsilon_0}\rho=\frac{\omega^2}{81}\left[\begin{aligned}&50\,\gamma_5^2\,\theta^{-2/3}+\left(630\,\gamma_5\gamma_7+324\right)\theta^{0}\\&+\left(980\,\gamma_7^2+1.800\,\gamma_5\gamma_9-1.440\,\gamma_5\right)\theta^{2/3}\\&+\left(4.410\,\gamma_7\gamma_9+3.850\,\gamma_5\gamma_{11}+1.710\,\gamma_5^2-2.520\,\gamma_7\right)\theta^{4/3}\\&+\cdots\end{aligned}\right.$$

and the component of the current density :

$$-\frac{\eta}{\varepsilon_0}j_r=-\frac{\eta}{\varepsilon_0}\rho V_r=\frac{r\omega^3}{81}\left[\begin{aligned}&-100\,\gamma_5^2\,\theta^{1/3}\\&+\left(100\,\gamma_5^3-1260\,\gamma_5\gamma_7-648\right)\theta^{3/3}\\&+\cdots\end{aligned}\right.$$

$$-\frac{\eta}{\varepsilon_0}j_\theta=-\frac{\eta}{\varepsilon_0}\rho V_\theta=\frac{r\omega^3}{243}\left[\begin{aligned}&250\,\gamma_5^3\,\theta^{0}\\&+\left(3.500\,\gamma_5^2\gamma_7+1.620\,\gamma_5\right)\theta^{2/3}\\&+\left(\begin{aligned}&9.450\,\gamma_5^2\gamma_9+9.310\,\gamma_5\gamma_7^2\\&+2.268\,\gamma_7-7.200\,\gamma_5^2\end{aligned}\right)\theta^{4/3}\\&+\left(\begin{aligned}&19.800\,\gamma_5^2\gamma_{11}+40.320\,\gamma_5\gamma_7\gamma_9\\&+6.860\,\gamma_7^3+2.916\,\gamma_9\\&-22.680\,\gamma_5\gamma_7+8.550\,\gamma_5^3\end{aligned}\right)\theta^{6/3}\\&+\cdots\end{aligned}\right.$$

The second expression exhibits the meaning of the constant γ_5. At the cathode the current density is J .

$$J = -\frac{\varepsilon_0}{\eta} \times \frac{250}{243} \times \gamma_5^3 \, r \, \omega^3$$

J is proportionnal to r, and to the cube of the Larmor angular frequency an to γ_5

$\mathrm{Div}\, j = 0$ gives

$$-\frac{\eta}{\varepsilon_0} \times \frac{1}{r} \times \frac{\partial}{\partial r}(j_r r) = \frac{2\omega^3}{81}\left[-100\gamma_5^2\theta^{1/3} + (100\gamma_5^3 - 1260\gamma_5\gamma_7 - 648)\theta^{3/3} + \dots\right]$$

$$-\frac{\eta}{\varepsilon_0} \times \frac{1}{r} \times \frac{\partial}{\partial \theta}(j_\theta) = \frac{2\omega^3}{729}\left[\begin{aligned} &(3.500\gamma_5^2\gamma_7 + 1.620\gamma_5)\theta^{-1/3} \\ &\left(\begin{aligned} &+18.900\gamma_5^2\gamma_9 + 18620\gamma_5\gamma_7^2 \\ &+4536\gamma_7 - 14400\gamma_5 \end{aligned}\right)\theta^{1/3} \\ &\left(\begin{aligned} &+59.400\gamma_5^2\gamma_{11} + 120960\gamma_5\gamma_7\gamma_9 \\ &+20580\gamma_7^3 + 8748\gamma_9 \\ &-68040\gamma_5\gamma_7 + 25650\gamma_5^3 \end{aligned}\right)\theta^{3/3} \\ &+ \dots \end{aligned}\right.$$

And the comparison of the coefficients

$$\theta^{-1/3} : \quad 3500\,\gamma_5^2\,\gamma_7 + 1620\,\gamma_5 = 0$$

$$\gamma_7 = -\frac{81}{175} \times \frac{1}{\gamma_5}$$

$$\theta^{+1/3} : \quad 18.900\,\gamma_5^2\,\gamma_9 + 18.620\,\gamma_5\,\gamma_7^2 + 4.536\,\gamma_7 - 15.300\,\gamma_5^2 = 0$$

$$\gamma_9 = \frac{17}{21} + \frac{6}{25} \times \frac{81}{175} \times \frac{1}{\gamma_5^3} - \frac{133}{135} \times \frac{\overline{81}^2}{\overline{175}^2} \times \frac{1}{\gamma_5^3}$$

$$= \frac{17}{21} - \frac{2.187}{21.875} \times \frac{1}{\gamma_5^3}$$

$$\theta^{+1} : \quad \left.\begin{aligned} &59.400\,\gamma_5^2\,\gamma_{11} + 120.960\,\gamma_5\,\gamma_7\,\gamma_9 + 20.580\,\gamma_7^3 \\ &+ 8.748\,\gamma_9 - 79.380\,\gamma_5\,\gamma_7 + 26.550\,\gamma_5^3 - 5.832 \end{aligned}\right\} = 0$$

$$\gamma_{11} = \left[\begin{aligned} &- \frac{59}{132}\,\gamma_5 + \frac{27}{275} \cdot \frac{1}{\gamma_5^2} - \frac{147}{110} \cdot \frac{81}{175} \cdot \frac{1}{\gamma_5^2} \\ &- \frac{81}{550}\left(\frac{17}{21} - \frac{2.187}{21.875} \times \frac{1}{\gamma_5^3}\right)\frac{1}{\gamma_5^2} \\ &+ \frac{112}{55} \times \frac{81}{175}\left(\frac{17}{21} - \frac{2.187}{21.875} \times \frac{1}{\gamma_5^3}\right)\frac{1}{\gamma_5^2} \\ &+ \frac{343}{990} \times \frac{\overline{81}^3}{\overline{175}^3} \times \frac{1}{\gamma_5^5} \end{aligned}\right.$$

$$\gamma_{11} = -\frac{59}{132}\,\gamma_5 + \frac{108}{875} \times \frac{1}{\gamma_5^2} - \frac{1.358.127}{30.078.125} \times \frac{1}{\gamma_5^5}$$

γ_5	0.1	0.2	0.5	1	2	5	10
$\gamma_7 = -\frac{81}{175} \cdot \frac{1}{\gamma_5}$	$-4,629$	$-2,315$	$-0,9258$	$-0,4629$	$-0,2315$	$-0,09258$	$-0,04629$
$\frac{17}{21}$	$0,8095$	$0,8095$	$0,8095$	$0,8095$	$0,8095$	$0,8095$	$0,8095$
$-\frac{2187}{21875} \cdot \frac{1}{\gamma_5^3}$	$-99,98$	$-12,498$	$-0,7998$	$-0,0999$	$-1,25 . 10^{-2}$	$-8 . 10^{-4}$	$-9,998 . 10^{-8}$
γ_9	$-99,17$	$-11,69$	$+0,1097$	$+0,7095$	$+0,7970$	$+0,8087$	$+0,8094$
$-\frac{59}{132}\gamma_5$	$-0,0447$	$-0,0894$	$-0,2235$	$-0,447$	$-0,894$	$-2,235$	$-4,47$
$+\frac{108}{875} \cdot \frac{1}{\gamma_5^2}$	$12,34$	$3,085$	$+0,4936$	$+0,1234$	$+0,03085$	$4,93 . 10^{-3}$	$1,23 . 10^{-3}$
$-0,045153 \cdot \frac{1}{\gamma_5^5}$	-4515	$-141,09$	$-1,445$	$-0,04515$	$-1,411 . 10^{-3}$	$-1,445 . 10^{-5}$	$-4,515 . 10^{-7}$
γ_{11}	-4503	$-138,1$	$-1,175$	$-0,3688$	$-0,8646$	$-2,23$	$-4,469$

Calculation of $f(\Theta)$ and of its deviations for $\Theta = 10^{-3}$

$\gamma_5\ \theta^{5/3}$	10^{-6}	2×10^{-6}	5×10^{-6}	10^{-5}	2×10^{-5}	5×10^{-5}
$\gamma_7\ \theta^{7/3}$	$-4{,}629 \cdot 10^{-7}$	$-2{,}315 \times 10^{-7}$	$-9{,}26 \times 10^{-8}$	$-4{,}629 \times 10^{-8}$		
$\gamma_9\ \theta^{9/3}$	$-9{,}998 \cdot 10^{-8}$	$-1{,}169 \times 10^{-8}$	$+0{,}011 \times 10^{-8}$	$+0{,}071 \times 10^{-8}$		
$\gamma_{11}\ \theta^{11/3}$	$-4{,}503 \times 10^{-8}$	$-0{,}138 \times 10^{-8}$	$-0{,}0001 \times 10^{-8}$			
f	$(39{,}2 \pm 5)10^{-8}$	$(175{,}5 \pm 0{,}2)10^{-8}$	$490{,}7 \times 10^{-8}$	$995{,}4 \times 10^{-8}$	$997{,}7 \times 10^{-8}$	4999×10^{-8}

γ_5	0,1	0,2	0,5	1,0	2,0	5,0	10,0
$5/3\ \gamma_5\ \theta^{2/3}$	$1,667 \cdot 10^{-3}$	$3,333 \cdot 10^{-3}$	$8,333 \cdot 10^{-3}$	$1,667 \cdot 10^{-2}$	$3,333 \cdot 10^{-2}$	$8,333 \cdot 10^{-2}$	$1,667 \cdot 10^{-1}$
$7/3\ \gamma_7\ \theta^{4/3}$	$-1,08 \cdot 10^{-3}$	$-5,402 \cdot 10^{-4}$	$-2,15 \cdot 10^{-4}$	$-1,08 \cdot 10^{-4}$	$-5,402 \cdot 10^{-5}$	$-2,16 \cdot 10^{-5}$	$-1,08 \cdot 10^{-5}$
$9/3\ \gamma_9\ \theta^{6/3}$	$-2,975 \cdot 10^{-4}$	$-3,507 \cdot 10^{-5}$	$+3,291 \cdot 10^{-7}$	$+2,129 \cdot 10^{-6}$	$2,391 \cdot 10^{-6}$	$2,426 \cdot 10^{-6}$	$2,428 \cdot 10^{-6}$
$11/3\ \gamma_{11}\ \theta^{8/3}$	$-1,653 \cdot 10^{-4}$	$-5,063 \cdot 10^{-6}$	$-4,308 \cdot 10^{-8}$	$-1,352 \cdot 10^{-8}$	$-3,170 \cdot 10^{-8}$	$-8,177 \cdot 10^{-8}$	$-1,639 \cdot 10^{-7}$
f^{I}	$(1,24 \pm 1)\, 10^{-4}$	$(2,753 \pm 0,05) \cdot 10^{-3}$	$8,117 \cdot 10^{-3}$	$1,656 \cdot 10^{-2}$	$3,328 \cdot 10^{-2}$	$8,331 \cdot 10^{-2}$	$1,667 \cdot 10^{-1}$
$10/9\ \gamma_5\ \theta^{-1/3}$	1,111	2,222	5,555	11,111	22,222	55,55	111,11
$28/9\ \gamma_7\ \theta^{1/3}$	-1,44	-0,72	-0,288	-0,144	-0,072	-0,0288	-0,0144
$54/9\ \gamma_9\ \theta^{3/3}$	-0,595	-0,0714	-0,0007	-0,0043	-0,0048	-0,0049	-0,0049
$88/9\ \gamma_{11}\ \theta^{5/3}$	-0,4403	-0,0135	$-1,199 \cdot 10^{-4}$	–	–	–	–
f^{II}		1,42 ± 0,01	5,266	10,96	22,14	55,52	111,1
$-\frac{10}{27}\ \gamma_5\ \theta^{-4/3}$	-370,4	-740,7	-1851,8	-3703,1	-7407,4	-18518,5	-37037
$+\frac{28}{27}\ \gamma_7\ \theta^{-2/3}$	-480	-240	-96	-48	-24	-9,6	-4,8
$+\frac{162}{27}\ \gamma_9\ \theta^{0}$	-595	-71,4	-0,658	–	–	–	–
$+\frac{440}{27}\ \gamma_{11}\ \theta^{2/3}$	-733,8	-22,49	-0,1915	-0,0601	-0,1409	-0,3634	-0,7283
f^{III}		-1074 ± 2	-1949	-3752	-7431	-18529	-37.086
$+\frac{40}{81}\ \gamma_5\ \theta^{-7/3}$	$4,938 \cdot 10^{+5}$	$9,876 \cdot 10^{5}$	$2,469 \cdot 10^{6}$	$4,938 \cdot 10^{6}$	$9,876 \cdot 10^{6}$	$2,469 \cdot 10^{7}$	$4,938 \cdot 10^{7}$
$-\frac{56}{81}\ \gamma_7\ \theta^{-5/3}$	$3,200 \cdot 10^{5}$	$1,6 \cdot 10^{5}$	$6,4 \cdot 10^{4}$	$3,2 \cdot 10^{4}$	$1,6 \cdot 10^{4}$	$6,4 \cdot 10^{3}$	$3,2 \cdot 10^{3}$
$+\frac{880}{81}\ \gamma_{11}\ \theta^{-1/3}$	$-5,001 \cdot 10^{5}$	$-1,5 \cdot 10^{4}$	$-1,277 \cdot 10^{2}$	-40,07	–	–	–
f^{IV}		$(113 \pm 2)\, 10^{4}$	$2,533 \cdot 10^{6}$	$4,97 \cdot 10^{6}$	$9,892 \cdot 10^{6}$	$2,47 \cdot 10^{7}$	$4,938 \cdot 10^{7}$

Equation of the trajectories.

$$\frac{dr}{r\,d\theta} = \frac{V_r}{V_\theta} = \frac{2(f-\theta)}{f'}$$

$$L\,\frac{r}{r_0} = 2\int_0^\theta \frac{f-\theta}{f'}\,d\theta$$

Evaluation for small values of θ

$$\frac{1}{f'} = \frac{3}{5\gamma_5}\,\theta^{-2/3}\left[1 + 1{,}4\,\frac{\gamma_7}{\gamma_5}\,\theta^{2/3} + 1{,}8\,\frac{\gamma_9}{\gamma_5}\,\theta^{4/3} + 2{,}2\,\frac{\gamma_{11}}{\gamma_5}\,\theta^{6/3} + \cdots\right]^{-1}$$

$$= \frac{3}{5\gamma_5}\,\theta^{-2/3}\left[1 - 1{,}4\,\frac{\gamma_7}{\gamma_5}\,\theta^{2/3} + \left(1{,}96\,\frac{\gamma_7^2}{\gamma_5^2} - 1{,}8\,\frac{\gamma_9}{\gamma_5}\right)\theta^{4/3} + \left(5{,}04\,\frac{\gamma_7\gamma_9}{\gamma_5^2} - 2{,}2\,\frac{\gamma_{11}}{\gamma_5}\right)\theta^{6/3} + \cdots\right.$$

$$\frac{f-\theta}{f'} = -\frac{3}{5\gamma_5}\,\theta^{1/3}\left[1 - 1{,}4\,\frac{\gamma_7}{\gamma_5}\,\theta^{2/3} + \left(1{,}96\,\frac{\gamma_7^2}{\gamma_5^2} - 1{,}8\,\frac{\gamma_9}{\gamma_5}\right)\theta^{4/3} + \left(5{,}04\,\frac{\gamma_7\gamma_9}{\gamma_5^2} - 2{,}2\,\frac{\gamma_{11}}{\gamma_5}\right)\theta^{6/3} + \cdots\right.$$

$$\times\left[1 - \gamma_5\,\theta^{2/3} - \gamma_7\,\theta^{4/3} - \gamma_9\,\theta^{6/3} + \cdots\right]$$

$$\frac{f-\theta}{f'} = -\frac{3}{5\gamma_5}\,\theta^{1/3}\left[1 - \left(\gamma_5 + 1{,}4\,\frac{\gamma_7}{\gamma_5}\right)\theta^{2/3} + \left(0{,}4\,\gamma_7 + 1{,}96\,\frac{\gamma_7^2}{\gamma_5^2} - 1{,}8\,\frac{\gamma_9}{\gamma_5}\right)\theta^{4/3} + \left(0{,}8\,\gamma_9 + 0{,}56\,\frac{\gamma_7^2}{\gamma_5} + 5{,}04\,\frac{\gamma_7\gamma_9}{\gamma_5^2} - 2{,}2\,\frac{\gamma_{11}}{\gamma_5}\right)\theta^{6/3} + \cdots\right.$$

$$L\frac{r}{r_0} = -\frac{9}{10\gamma_5}\,\theta^{1/3}\left[\begin{array}{l} 1-\frac{1}{3}\left(\gamma_5+1{,}4\frac{\gamma_7}{\gamma_5}\right)\theta^{2/3}+\left(0{,}2\gamma_7+0{,}98\frac{\gamma_7^2}{\gamma_5^2}-0{,}9\frac{\gamma_9}{\gamma_5}\right)\theta^{4/3} \\ +\left(0{,}32\gamma_9+0{,}224\frac{\gamma_7^2}{\gamma_5^2}+2{,}016\frac{\gamma_7\gamma_9}{\gamma_5^2}-0{,}88\frac{\gamma_{11}}{\gamma_5}\right)\theta^{6/3} \\ +\cdots \end{array}\right.$$

$$= -\frac{9}{10\gamma_5}\,\theta^{1/3}\left[\begin{array}{l} 1-\frac{\gamma_5}{3}\left(1-\frac{81}{125\,\gamma_5^3}\right)\theta^{2/3}+\frac{1}{\gamma_5}\left[-\frac{1437}{1750}+\frac{6561}{21875\,\gamma_5^3}\right]\theta^{4/3} \\ +\left[\frac{137}{210}-\frac{435256}{546875}\cdot\frac{1}{\gamma_5^3}+\frac{1299078}{9765625}\cdot\frac{1}{\gamma_5^6}\right]\theta^{6/3} \\ +\cdots \end{array}\right.$$

Expression for the potential

$$2\eta\,\Phi = \frac{25}{9}\,r^2\omega^2\gamma_5^2\,\theta^{4/3}\left[\begin{array}{l} 1+\frac{18}{125\,\gamma_5^2}\,\theta^{2/3} \\ +\frac{1}{\gamma_5}\left(\frac{6}{175}-\frac{6561}{109375}\cdot\frac{1}{\gamma_5^3}\right)\theta^{4/3} \\ +\left(\frac{472392}{13671875}\cdot\frac{1}{\gamma_5^6}-\frac{54}{4375}\cdot\frac{1}{\gamma_5^3}-\frac{29}{150}\right)\theta^{6/3} \\ +\cdots \end{array}\right.$$

Without magnetic field.

$$W = r^2 f$$

$$V_r = 2 r f$$

$$V_\theta = r f'$$

$$2\eta \Phi = r^2 \left(4 f^2 + f'^2\right)$$

$$\eta \Phi_r = r \left(4 f^2 + f'^2\right)$$

$$\eta \Phi_{rr} = 4 f^2 + f'^2$$

$$\frac{\eta}{r} \Phi_\theta = r \left(4 f f' + f' f''\right)$$

$$\frac{\eta}{r^2} \Phi_{\theta\theta} = 4 f'^2 + 4 f f'' + f''^2 + f' f'''$$

$$-\frac{\eta}{\varepsilon_0} \rho = 8 f^2 + 6 f'^2 + 4 f f'' + f''^2 + f' f'''$$

$$j_\theta = r \left[8 f^2 f' + 6 f'^3 + 4 f f' f'' + f' f''^2 + f'^2 f'''\right]$$

$$\frac{1}{r} \frac{\partial j_\theta}{\partial \theta} = 16 f f'^2 + 8 f^2 f'' + 18 f'^2 f'' + 4 f'^2 f'' + 4 f f''^2 + 4 f f' f''' + 2 f' f'' f''' + f''^3$$
$$+ 2 f' f'' f''' + f'^2 f^{IV}$$

$$j_r = r \left[16 f^3 + 12 f f'^2 + 8 f^2 f'' + 2 f f''^2 + 2 f f' f'''\right]$$

$$\frac{1}{r} \frac{\partial}{\partial r}\left(r j_r\right) = 32 f^3 + 24 f f'^2 + 16 f^2 f'' + 4 f f''^2 + 4 f f' f'''$$

$$\mathrm{div}\, j = 0$$

$$f''^3 + 32 f^3 + 40 f f'^2 + 24 f^2 f'' + 22 f'^2 f'' + 8 f f''^2 + 8 f f' f''' + f'^2 f^{IV} + 4 f' f'' f''' = 0$$

$$f = f_0\,\theta^{5/3}\left(1 + f_2\theta^2 + f_4\,\theta^4 + f_6\,\theta^6 + f_8\,\theta^8 + \cdots\right)$$

$$f' = \frac{1}{3}\,f_0\,\theta^{2/3}\left(5 + 11\,f_2\theta^2 + 17\,f_4\,\theta^4 + 23\,f_6\,\theta^6 + 29\,f_8\,\theta^6 + \cdots\right)$$

$$f'' = \frac{1}{9}\,f_0\,\theta^{-1/3}\left(10 + 88\,f_2\,\theta^2 + 238\,f_4\,\theta^4 + 460\,f_6\,\theta^6 + 754\,f_8\,\theta^8 + \cdots\right)$$

$$f''' = \frac{1}{27}\,f_0\,\theta^{-4/3}\left(-10 + 440\,f_2\,\theta^2 + 2618\,f_4\,\theta^4 + 7820\,f_6\,\theta^6 + 17342\,f_8\,\theta^8 + \cdots\right)$$

$$f^{IV} = \frac{1}{81}\,f_0\,\theta^{-7/3}\left(+40 + 880\,f_2\,\theta^2 + 20944\,f_4\,\theta^4 + 109.480\,f_6\,\theta^6 + 346.840\,f_8\,\theta^8 + \cdots\right)$$

The differential equation is written according to the powers of θ :

Power of the first term

$$0 = \begin{cases} f'^2 f^{IV} + 4\,f' f'' f''' + f''^3 & \theta^{-1} \\ 8\,f f''^2 + 8\,f f' f''' + 22\,f'^2 f'' & \theta^{+1} \\ 24\,f^2 f'' + 40\,f f'^2 & \theta^{3} \\ 32\,f^3 & \theta^{5} \end{cases}$$

For the first comparison of the coefficients (terms in θ^{-1}) we need the products of the first line.

$$f'^2 f^{IV} = \frac{f_0^3}{729}\,\theta^{-1}\Big[1000 + 26.400\,f_2\,\theta^2 + \left(530.400\,f_4 + 101.640\,f_2^2\right)\theta^4 + \left(2.745.200\,f_6 + 2.468.400\,f_2 f_4 + 106480\,f_2^3\right)\theta^6 + \cdots$$

$$4f'f''f''' = \frac{f_0^3}{729}\,\theta^{-1}\left[\begin{array}{l}-2000 + 66.000 f_2\theta^2 + (469.200 f_4 + 929.280 f_2^2)\theta^4 \\ + (1.462.800 f_6 + 7.988.640 f_2 f_4 + 1703680 f_2^3)\theta^6 \\ + \cdots \end{array}\right.$$

$$f''^3 = \frac{f_0^3}{729}\,\theta^{-1}\left[\begin{array}{l}1000 + 26.400 f_2\theta^2 + (71.400 f_4 + 232320 f_2^2)\theta^4 \\ + (256640 f_2 f_4 + 138000 f_6 + 681472 f_2^3)\theta^6 + \cdots \end{array}\right.$$

The comparison gives as expected

$$1000 + 1000 - 2000 = 0$$

For the second comparison (terms in θ^{+1}**) we need also the products**

$$8ff''^2 = \frac{f_0^3}{729}\,\theta^{-1}\left[\begin{array}{l}7200\theta^2 + 133920 f_2\theta^4 \\ + (684288 f_2^2 + 349920 f_4)\theta^6 + \cdots \end{array}\right.$$

$$8ff'f''' = \frac{f_0^3}{729}\,\theta^{-1}\left[-3600\theta^2 + 146880 f_2\theta^4 + (926640 f_4 + 498960 f_2)\theta^6 + \cdots\right]$$

$$22f'^2f'' = \frac{f_0^3}{729}\,\theta^{-1}\left[49.500\theta^2 + 653400 f_2\theta^4 + (1514700 f_4 + 2156220 f_2^2)\theta^6 + \cdots\right]$$

The determination in f_2 **gives**

$$25.400 f_2 + 66000 f_2 + 25400 f_2 + 7200 - 3500 + 49.500 = 0$$

$$f_2 = -\frac{59}{132} = -0,446969$$

For the third comparison (terms in θ^3) we need the products :

$$24\, f^2 f'' = \frac{f_0^3}{729}\, \theta^{-1} \left(19\,440.\theta^4 + 209952\, f_2 \theta^6 + \dots \right)$$

$$40\, f f'^2 = \frac{f_0}{729}\, \theta^{-1} \left(81.000\, \theta^4 + 437.400\, f_2 \theta^6 + \dots \right)$$

The determination of f_4 gives then

$$0 = \begin{cases} 530.400 f_4 + 101640\, f_2^2 + 469200\, f_4 \\ + 929280 f_2^2 + 71400\, f_4 + 232320\, f_2^2 + 146880 f_2 \\ + 653400\, f_2 + 19440 + 81000 \end{cases}$$

or

$$1.071.000 f_4 + 1263240\, f_2^2 + 934200\, f_2 + 100440 = 0$$

$$f_4 = -\frac{1}{1.071.000} \cdot \left[1263240 \cdot \left(\frac{59}{132}\right)^2 - 934.200 \cdot \frac{59}{132} + 100.440 \right]$$

$$f_4 = +\, 0{,}0604533$$

The determination of f_6 gives

$$0=\begin{cases} 2746200\, f_6 + 2468400\, f_4 f_2 + 106480\, f_2^3 \\ +1462800\, f_6 + 7988640\, f_4 f_2 + 1703680\, f_2^3 \\ +1256640\, f_4 f_2 + 138000\, f_6 + 681472\, f_2^3 \\ +(684288 + 498960 + 2156220)\, f_2^2 \\ +(349920 + 926640 + 1514700)\, f_4 \\ +(209952 + 437400)\, f_2 \\ +23328 \end{cases}$$

The last term is due to the cube $32f^3$ of the differential equation; it may be written

$$0=\begin{cases} 4347000\, f_6 + 11713680\, f_4 f_2 + 2491632\, f_2^3 \\ +3399468\, f_2^2 + 2791260\, f_4 + 647352\, f_2 + 23328 \end{cases}$$

$$f_6 = -0{,}007103308$$

Control :

$$V_r = 2rf = r f_0 \left(2\theta^{5/3} + 2 f_2 \theta^{11/3} + 2 f_4 \theta^{17/3} + 2 f_6 \theta^{23/3} + \cdots \right)$$

$$V_\theta = r f' = \frac{r f_0}{3} \left(5\theta^{2/3} + 11 f_2 \theta^{8/3} + 17 f_4 \theta^{14/3} + 23 f_6 \theta^{20/3} + \cdots \right)$$

$$V_r^2 = r^2 f_0^2 \left(4\theta^{10/3} + 8 f_2 \theta^{16/3} + (4 f_2^2 + 8 f_4)\theta^{22/3} + (8 f_2 f_4 + 8 f_6)\theta^{28/3} + \cdots \right)$$

$$V_\theta^2 = \frac{1}{9} r^2 f_0^2 \begin{pmatrix} 25\theta^{4/3} + 110 f_2 \theta^{10/3} + (121 f_2^2 + 170 f_4)\theta^{16/3} \\ + (374 f_2 f_4 + 230 f_6)\theta^{22/3} + \cdots \end{pmatrix}$$

$$2\eta\Phi = \frac{1}{9} r^2 f_0^2 \begin{pmatrix} 25\theta^{4/3} + (110 f_2 + 36)\theta^{6/3} + (121 f_2^2 + 170 f_4 + 72 f_2)\theta^{16/3} \\ + (374 f_2 f_4 + 230 f_6 + 36 f_2^2 + 72 f_4)\theta^{22/3} + \cdots \end{pmatrix}$$

$$\eta \frac{\partial\phi}{\partial r} = \frac{1}{9} r f_0^2 \left[\cdots \right]$$

$$\eta \frac{\partial^2\phi}{\partial r^2} = \frac{1}{9} f_0^2 \left[\cdots \right]$$

$$\frac{\eta}{r} \frac{\partial\phi}{\partial\theta} = \frac{r}{27} f_0^2 \begin{pmatrix} 50\theta^{1/3} + (550 f_2 + 180)\theta^{7/3} + (968 f_2^2 + 1360 f_4 + 576 f_2)\theta^{13/3} \\ + (4114 f_2 f_4 + 2530 f_6 + 396 f_2^2 + 792 f_4)\theta^{19/3} + \cdots \end{pmatrix}$$

$$\frac{\eta}{r^2} \frac{\partial^2\phi}{\partial\theta^2} = \frac{f_0^2}{81} \left[\begin{matrix} 50\theta^{-2/3} + (3850 f_2 + 1260)\theta^{4/3} + (12584 f_2^2 + 17680 f_4)\theta^{10/3} \\ + 7488 f_2 \theta^{10/3} + (78166 f_2 f_4 + 48070 f_6 + 7524 f_2^2)\theta^{16/3} \\ + 15048 f_4 \theta^{16/3} + \cdots \end{matrix} \right.$$

$$-\frac{\eta}{\varepsilon_0}\rho = \frac{f_0^2}{81} \left[\begin{matrix} 50\theta^{-2/3} + (3850 f_2 + 1710)\theta^{4/3} \\ + (12584 f_2^2 + 17680 f_4 + 9468 f_2 + 648)\theta^{10/3} \\ + (78166 f_2 f_4 + 48070 f_6 + 9702 f_2^2 + 18108 f_4 \\ + 1296 f_2)\theta^{16/3} + \cdots \end{matrix} \right.$$

$$\frac{\eta}{\varepsilon_0} j_\theta = \frac{r f_0^3}{243}\Big[250 + (19800 f_2 + 8550)\theta^2 + (105270 f_2^2 + 89250 f_4 + 66150 f_2 + 3240)\theta^4 + (650760 f_2 f_4 + 241500 f_6 + 152658 f_2^2 + 119610 f_4 + 13608 f_2 + 138424 f_2^3)\theta^6 + \cdots$$

$$\frac{\eta}{\varepsilon_0}\frac{\partial}{\partial\theta} j_\theta = \frac{2 r f_0^3}{243}\Big[(19800 f_2 + 8550)\theta + (210540 f_2^2 + 178500 f_4 + 6480)\theta^3 + (1952280 f_2 f_4 + 724500 f_6 + 457974 f_2^2 + 358830 f_4 + 40824 f_2 + 415272 f_2^3)\theta^5 + \cdots$$

$$\frac{\eta}{\varepsilon_0} j_r = \frac{2 r f_0^3}{81}\Big[50\theta + (3900 f_2 + 1710)\theta^3 + (16434 f_2^2 + 17730 f_4 + 11178 f_2 + 648)\theta^5 + \cdots$$

$$\frac{\eta}{\varepsilon_0}\frac{1}{r}\frac{\partial}{\partial r}(r j_r) = \frac{2 f_0}{81}\Big[100\theta + (7800 f_2 + 3420)\theta^3 + (32868 f_2^2 + 35460 f_4 + 22356 f_2 + 1296)\theta^5 + \cdots$$

$$\operatorname{div} j = 0$$

Comparison of the coefficient :

$\underline{\underline{f_2}}$ $\quad 19800 f_2 + 8550 + 300 = 0 \qquad f_2 = -0{,}446969$

$\underline{\underline{f_4}}$ $\quad \left.\begin{matrix} 210540 f_2^2 + 178500 f_4 + 132300 f_2 \\ + 6480 + 23400 f_2 + 10260 \end{matrix}\right\} = 0 \qquad f_4 = +0{,}06045433$

$\underline{\underline{f_6}}$ $\quad \left.\begin{matrix} 650760 f_2 f_4 + 241500 f_6 + 152658 f_2^2 \\ + 119610 f_4 + 13608 f_2 + 32868 f_2^2 \\ + 35460 f_4 + 22356 f_2 + 138424 f_2^3 \\ + 1296 \end{matrix}\right\} = 0 \qquad f_6 = -0{,}00710331$

For f_o, we get

$$\frac{\eta}{\varepsilon_o} j_o = \frac{250}{243} r_o f_o^3$$

$$f_o = \sqrt[3]{\frac{1,76 \cdot 10^{11} \times 243}{8,85 \cdot 10^{-12} \times 250}} \cdot \sqrt[3]{\frac{j_o[A/m^2]}{r_o[m]}} = 2,68 \times 10^5 \cdot \sqrt[3]{\frac{j_o[A/cm^2]}{r_o[cm]}}$$

Example

$$j_o = 1\,A/cm^2 \qquad r_o = 8\,cm$$

$$f_o = 1,34 \times 10^5\, A^{1/3} cm$$

Equation of the trajectories

$$L\frac{r}{r_o} = 2\int_0^\theta \frac{f\,d\theta}{f'}$$

$$f = f_o\,\theta^{5/3}\left(1 - 0,446970\,\theta^2 + 0,0604543\,\theta^4 - 0,0071033\,1\theta^6 + \dots\right)$$

$$f' = \frac{5}{3} f_o\,\theta^{2/3}\left(1 - 0,983333\,\theta^2 + 0,205545\,\theta^4 - 0,0326752\,\theta^6 + \dots\right)$$

$$\frac{1}{f'} = \frac{3}{5}\,\frac{\theta^{-2/3}}{f_o}\left(\begin{array}{l}1 + 0,983333\,\theta^2 + (0,966944 - 0,205545)\,\theta^4 \\ + (0,0326752 + 0,950828 - 0,404238)\,\theta^6 + \dots\end{array}\right.$$

$$\frac{2f'}{f} = 1,2\,\theta\left(1 + 0,536363\,\theta^2 + 0,38233\,\theta^4 + 0,291287\,\theta^6 + \dots\right)$$

$$L\frac{r}{r_o} = 0,6\,\theta^2\left(1 + 0,26818\,\theta^2 + 0,12744\,\theta^4 + 0,07282\,\theta^6 + \dots\right)$$

θ	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
$\frac{r}{r_o}$	1,006	1,024	1,057	1,105	1,175	1,272	1,41	1,61	1,92	2,42

Let us seek for the value of θ for which V_θ is zero

$$V_\theta = r f' = 0 \qquad f' = 0 \qquad f = \frac{V_r}{2r} \neq 0 \qquad f'' \neq 0$$

The differential equation gives then

$$f''^3 + 8ff''^2 + 24f^2f'' + 32f_3^3 = 0$$

The only real root of this equation of the third degree for f'' is

$$f'' = -4f$$

Since

$$L\frac{r}{r_0} = 2\int_0^\theta \frac{f\,d\theta}{f'}$$

r becomes infinite when $f' \to 0$

So that an asymptote exists. Along this asymptote, we have

$$\frac{\eta}{\varepsilon_0} j_r = 16rf^3 \qquad 2\eta\Phi = 4r^2f^2 \qquad j_\theta = 0 \qquad j_r = \frac{4\varepsilon_0\sqrt{2\eta}}{r^2}\Phi^{3/2}$$

Let us call θ_0 the angle of the asymptote and introduce a new variable $\delta = \theta_0 - \theta$ Since $f' = f''' = 0$ $f'' \neq 0$

We write

$$f = g_0 + g_2\delta^2 + g_4\delta^4 + g_6\delta^6 + \cdots$$

$$f' = 2g_2\delta + 4g_4\delta^3 + 6g_6\delta^5 + \cdots$$

$$f'' = 2g_2 + 12g_4\delta^2 + 30g_6\delta^4 + \cdots$$

$$f''' = 24g_4\delta + 120g_6\delta^3 + \cdots$$

$$f^{IV} = 24g_4 + 360g_6\delta^2 + \cdots$$

$$f'^2 f^{IV} = 96\, g_4 g_2^2 \delta^2 + (384\, g_2 g_4^2 + 1440\, g_2^2 g_6)\, \delta^4$$

$$4 f' f'' f''' = 384\, g_2^2 g_4\, \delta^2 + (3456\, g_2 g_4^2 + 1920\, g_2^2 g_6)\, \delta^4$$

$$f''^3 = 8\, g_2^3 + 144\, g_2^2 g_4\, \delta^2 + (864\, g_2 g_4^2 + 360\, g_2^2 g_6)\, \delta^4$$

$$\begin{aligned} 8 f f''^2 = {} & 32\, g_0 g_2^2 + (32\, g_2^3 + 384\, g_0 g_2 g_4)\, \delta^2 \\ & + (1152\, g_0 g_4^2 + 960\, g_0 g_2 g_6 + 416\, g_2^2 g_4)\, \delta^4 + \cdots \end{aligned}$$

$$8 f f' f''' = 384\, g_0 g_2 g_4\, \delta^2 + (384\, g_2^2 g_4 + 786\, g_4^2 g_0 + 960\, g_0 g_2 g_6)\, \delta^4$$

$$22 f'^2 f'' = 176\, g_2^3 \delta^2 + 1760\, g_2^2 g_4\, \delta^4$$

$$\begin{aligned} 24 f^2 f'' = {} & 48\, g_0^2 g_2 + (288\, g_0^2 g_4 + 96\, g_0 g_2^2)\, \delta^2 + \\ & + (48\, g_2^3 + 96\, g_0 g_2 g_4 + 576\, g_0 g_2 g_4 + 720\, g_0^2 g_6)\, \delta^4 + \cdots \end{aligned}$$

$$40 f f'^2 = 160\, g_0 g_2^2\, \delta^2 + (640\, g_0 g_2 g_4 + 160\, g_2^3)\, \delta^4$$

$$32 f^3 = 32\, g_0^3 + 96\, g_0^2 g_2\, \delta^2 + (96\, g_0 g_2^2 + 96\, g_0^2 g_2)\, \delta^4$$

Comparison of the coefficient

δ^0 $\quad g_2 = -2g_0$

δ^2 $\quad g_4 = -\frac{32}{39} g_0$

δ^4 $\quad g_6 =$

Trajectories

$$L\,\frac{r}{r_0} = 2\int_0^\theta \frac{f}{f'}\,d\theta$$

$$\frac{f}{f'} = \frac{\delta^{-1}}{2g_2}\left[1 + 2\frac{g_4}{g_2}\delta^2 + 3\frac{g_6}{g_2}\delta^4 + \cdots\right] = \frac{\delta^{-1}}{2g_2}\left[1 - 2\frac{g_4}{g_2}\delta^2 + \left(4\frac{g_4^2}{g_2^2} - 3\frac{g_6}{g_2}\right)\delta^4 + \cdots\right]$$

$$\frac{2f'}{f} = \frac{\delta^{-1}g_0}{g_2}\left[1 + \left(\frac{g_2}{g_0} - 2\frac{g_4}{g_2}\right)\delta^2 + \left(4\frac{g_4^2}{g_2^2} - \frac{g_4}{g_0} - 3\frac{g_6}{g_0}\right)\delta^4 + \cdots\right]$$

$$L\frac{r}{r_0} = \frac{g_0}{g_2}L\delta + \frac{1}{2}\left(1 - 2\frac{g_4 g_0}{g_2^2}\right)\delta^2 + \frac{1}{4}\left(\frac{g_4^2 g_0}{g_2^2} - \frac{g_4}{g_2} - 3\frac{g_6 g_0}{g_2^2}\right)\delta^4 + \cdots$$

$$L\frac{r}{r_0} = -\frac{1}{2}L\delta + \frac{1}{2}\left(1 - \frac{16}{39}\right)\delta^2 + \cdots = -\frac{1}{2}L\,\delta\,e^{-\frac{23}{39}\delta^2 + \cdots}$$

$$\frac{r_0^2}{r^2} = \delta\,e^{-\frac{23}{39}\delta^2 + \cdots}$$

The last point computed in the table $\frac{r}{r_0} = f(\theta)$ was $\frac{r}{r_0} = 2{,}42 \,\|\, \theta = 1$ Let us introduce this value in the above equation

$$\frac{1}{2{,}42^2} = \delta\,e^{-\frac{23}{39}\delta^2 + \cdots}$$

Result

$$\delta = 0{,}17$$

So that the angle of the asymptote is $\theta_0 = \theta_1 + \delta_2 = 1 + 0{,}17 = 1{,}17$

$$\theta_0 \approx 67^\circ$$

In the region free of space charge we have the Laplace's equation

$$\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r}\frac{\partial \phi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \phi}{\partial r^2} = 0$$

Along the asymptote :

$$\Phi = \frac{2 r^2}{\eta} g_0^2 \qquad \Phi' = 0$$

Let us assume for Φ a law :

$$\Phi = A r^2 h(\delta)$$

The Laplace's equation gives th en

$$4h + h'' = 0$$

And, with the initial conditions

$$\Phi = A r^2 \cos 2\delta = \frac{2 g_0^2}{\eta} r^2 \cos 2\delta$$

This field may be realized by two electrodes one of them being planar

$$\theta = C = \theta_0 + 45^\circ \approx 112^\circ$$

and the other an hyperbolic cylinder

$$r^2 (\cos^2\delta - \sin^2\delta) = C$$

INTERPRETATION OF THE RESULTS.

Let us consider at first the case where B = 0, the figure 18 shows how a convergent flat beam could be built; the approximate shape of the electrodes is shown in the figure 24 (b). The current density of the left hand side gun is highly non uniform since it is zero at the angle and a more uniform density should be obtained with the right hand side gun.

The figures 19 and 20 give the trajectories obtained for various magnetic field the factor F_0 being related to the magnetic field by

$$F_0^3 = \frac{250}{243} \frac{\eta}{\varepsilon_0} \frac{J}{r\omega^3}$$

ω being the Larmor's angular frequency $\eta B/2$.

To give a physical meaning to these solutions we must consider the cathode as a double sheet plane; the initial velocity is zero but the velocity for $\Theta = 360^\circ$ is never zero. (Fig.24 (a)).

A very unexpected result is the singular points which appear for $F_0 = 1.025 - 0.884 - 0.746$ and probably others for higher magnetic fields which have not been computed. For these values a zero velocity - zero potential point occurs at angles of respectively 132° - 300° - 218°. It may be considered that these values are interpolated; the accuracy is about $\pm$ 0°5.

The radius is unity at this point and a symetry exists between this virtual cathode and the original cathode. This is shown with more details in the figure 21. The particular case F_0 = 1.026 is more carefully studied with an equipotential; the equipotential corresponds to $\frac{2\eta\phi}{\omega^2} = 1$ (all the equipotentials in the beam are homothetical).

This case is of interest for the study of the noise because it is a well defined system in which space charge oscillations could occur; the symetry may simplify the solution of the equation of perturbation.

A practical such gun could be built with suitable electrodes; they have not been yet computed and the sketch of the figure 24 (c) is only an approximation.

If the beam reach just the anode (critical magnetic field). We have

$$\frac{2\eta\Phi}{\omega^2} = 1.1$$

$$\frac{\eta J}{\varepsilon_0 r \omega^3} = 0{,}97 \times 1{,}026 = 1$$

The total current is $I = \frac{J/r}{2}$

so that

$$Z = \frac{V}{I} \frac{\eta B/c}{\sqrt{\mu_0/\varepsilon}} = 2.2$$

instead of 1 for a classical brillouin beam.

A double mode operation(figure 24(d)) could be considered also.

For magnetic fields a little smaller than the magnetic field corresponding to the critical points, highly convergent beams can be obtained since the asymptote of the trajectory goes near the pole.

This is the case around 0.715; suitable electrodes have to be computed; such a gun is shown in the figure 24(c).

-:-:-

LIST OF FIGURES

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16 Gridded gun; maximum grid potential without sole current versus the line voltage. Sole current without grid B = 250 Gauss.

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18 Non uniform gun. Trajectory without magnetic field.

19 $F_o = \infty$ 10 5 2 1.4 1.3 1.2 1.1 1.05 1 0.95 0.9.

20 F_o = 0.9 0.885 0.85 0.8 0.75 0.7 0.65 0.6 0.55 0.5

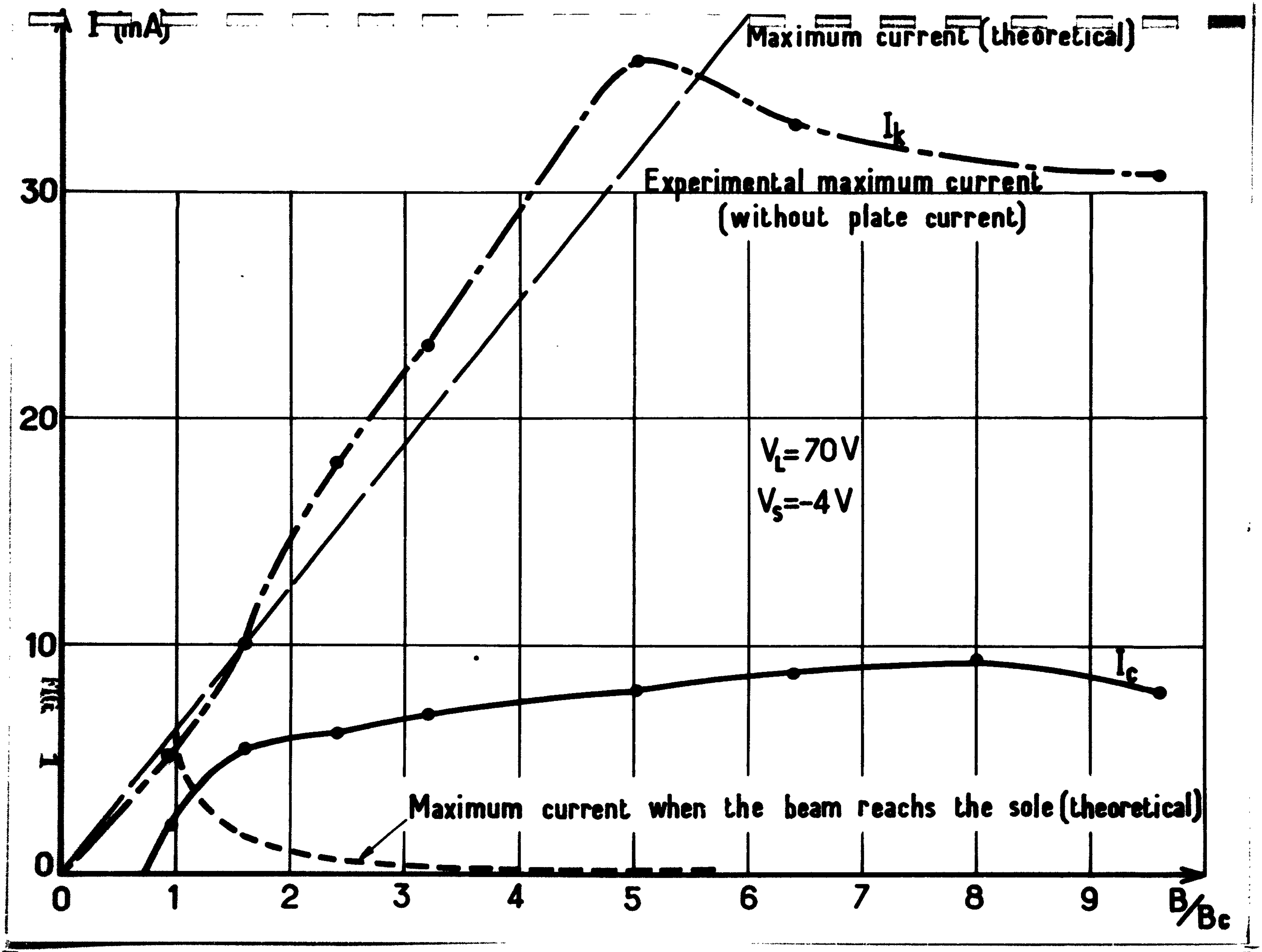
21 F_o = (I 1.1 1.09 1.08 1.07 1.06 1.05 1.04 1.03 1.02 1.01 1
(II 0.895 0.89 0.885 0.88 0.875 0.87 0.865 0.86.
(III 0.75 0.745 0.740 0.735 0.730 0.725 0.720 0.715 0.710 0.705 0.7

22 F_o = 1.025 trajectory and equipotential.

23 F_o = 0.500 0.505 0.510 0.515 0.520 0.530 0.540.

24 Schetch of guns.

-:-:-



I (mA)
Maximum current (theoretical)
I_k
Experimental maximum current
(without plate current)
V_L=70 V
V_s=-4 V
I_c
Maximum current when the beam reachs the sole (theoretical)
0
10
20
30
1
2
3
4
5
6
7
8
9
B/Bc

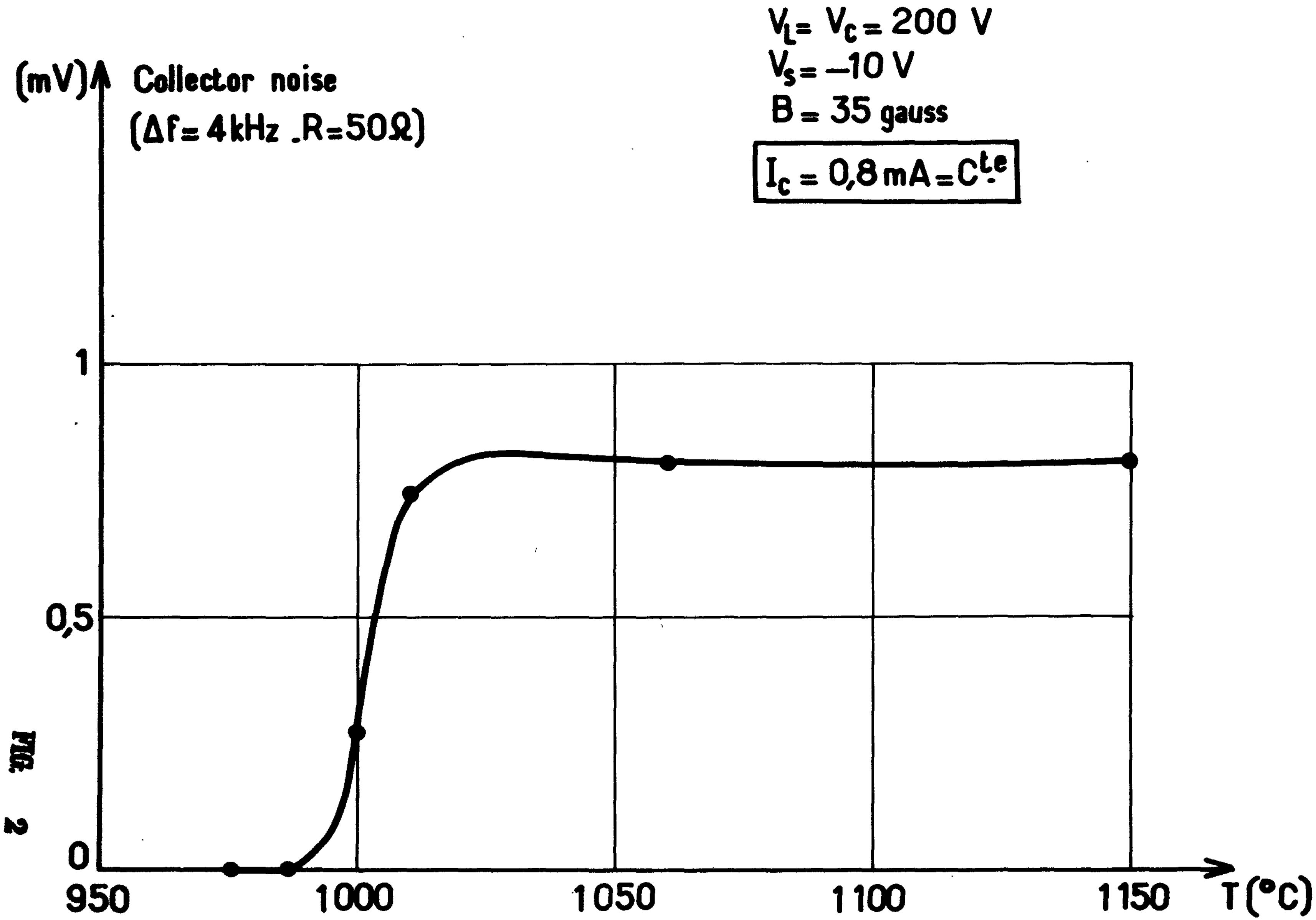

(mV)
Collector noise
(Δf = 4 kHz . R = 50Ω)
$V_L = V_C = 200$ V
$V_s = -10$ V
B = 35 gauss
$I_c = 0{,}8$ mA = C^{te}
1
0,5
0
950
1000
1050
1100
1150
T (°C)

FIG. 2

NOISE SIGNAL

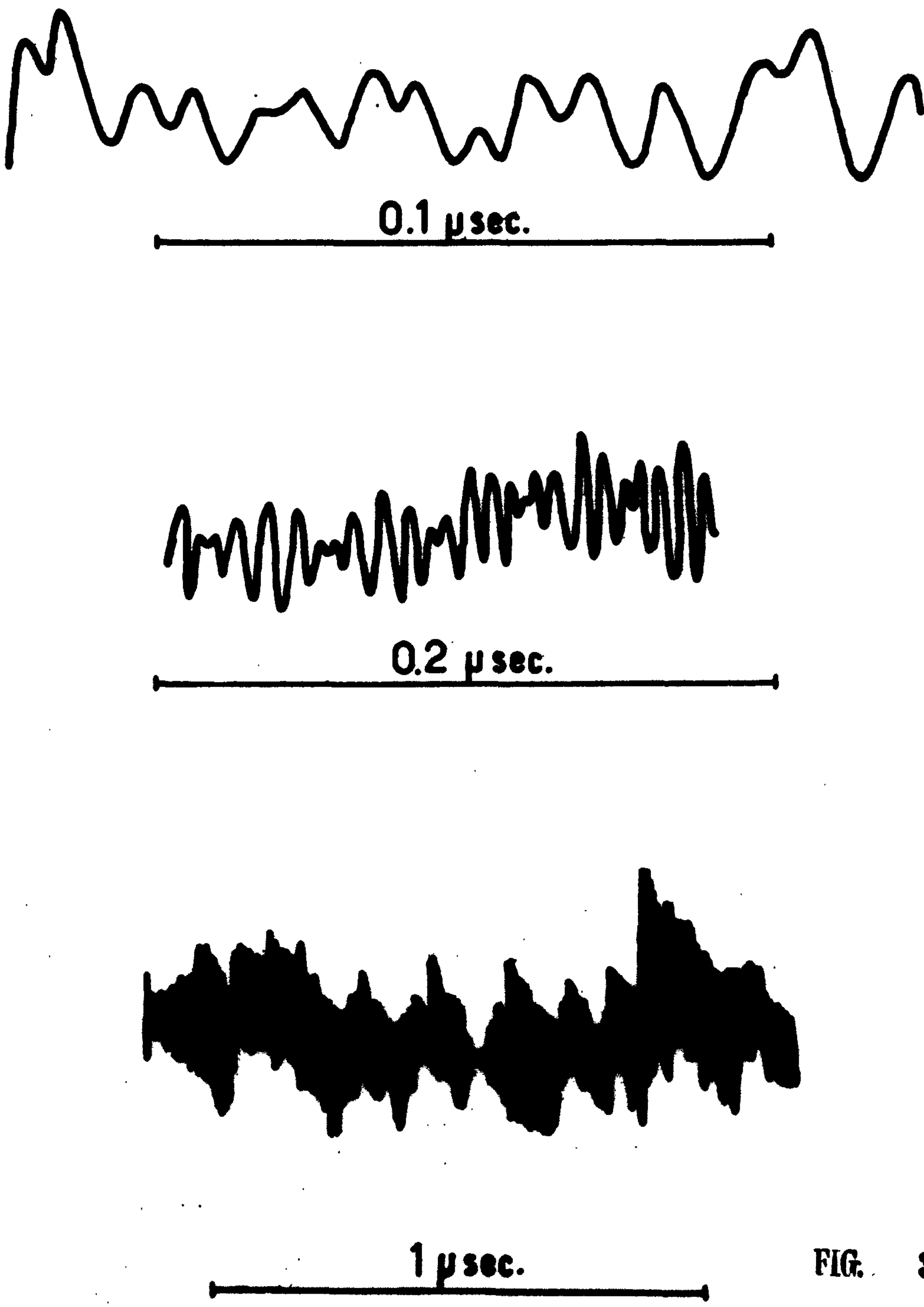
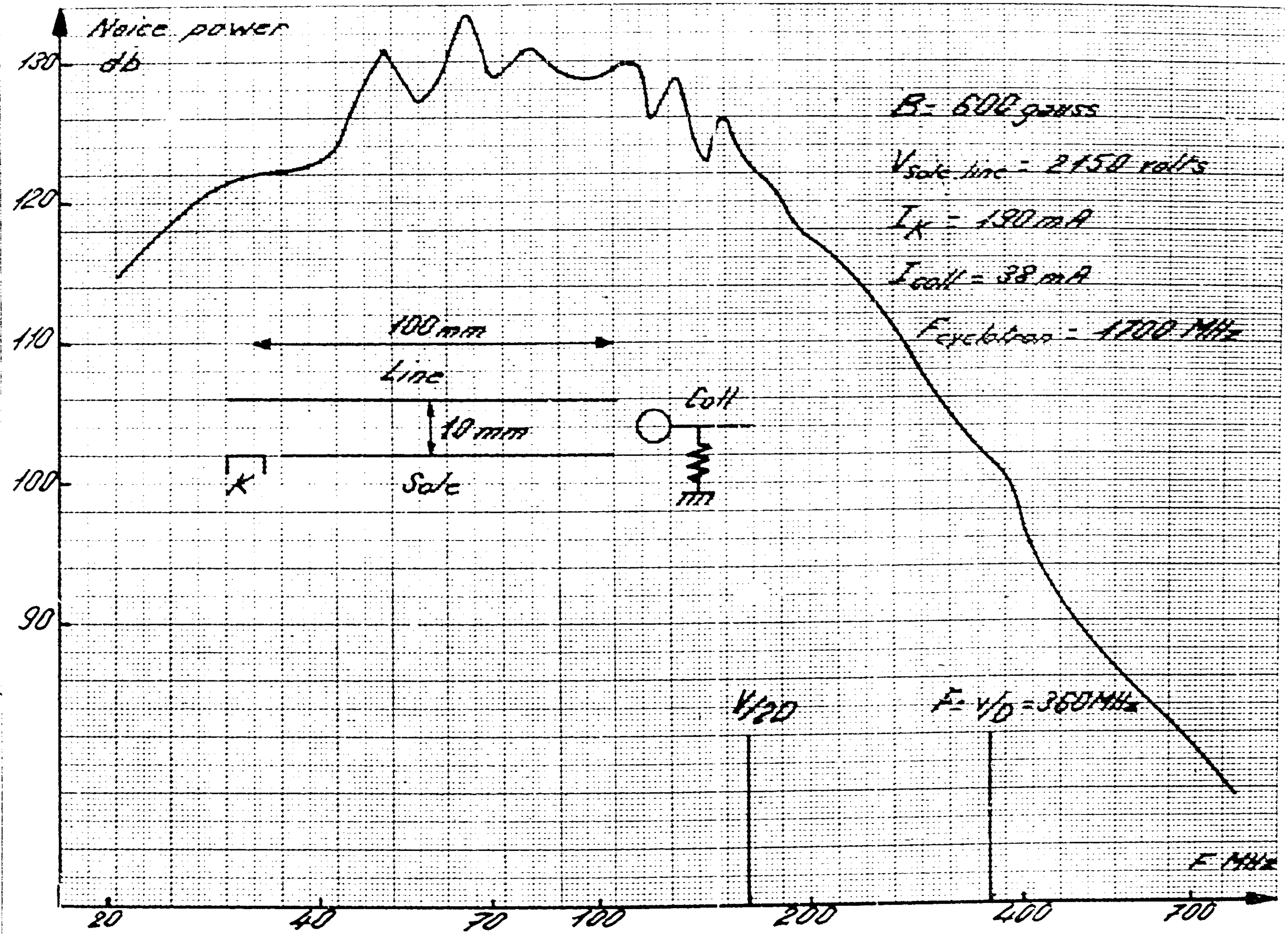


FIG. 8

CSF



Noice power
db
130
120
110
100
90
20
40
70
100
200
400
700
F MHz
B = 600 gauss
$V_{sole\ line}$ = 2150 volts
I_K = 190 mA
I_{coll} = 38 mA
$F_{cyclotron}$ = 1700 MHz
100 mm
Line
10 mm
Sole
K
Coll
V/2D
F = V/D = 360 MHz

Fig. 4

SCALING LAW IN PLANAR OPTICS

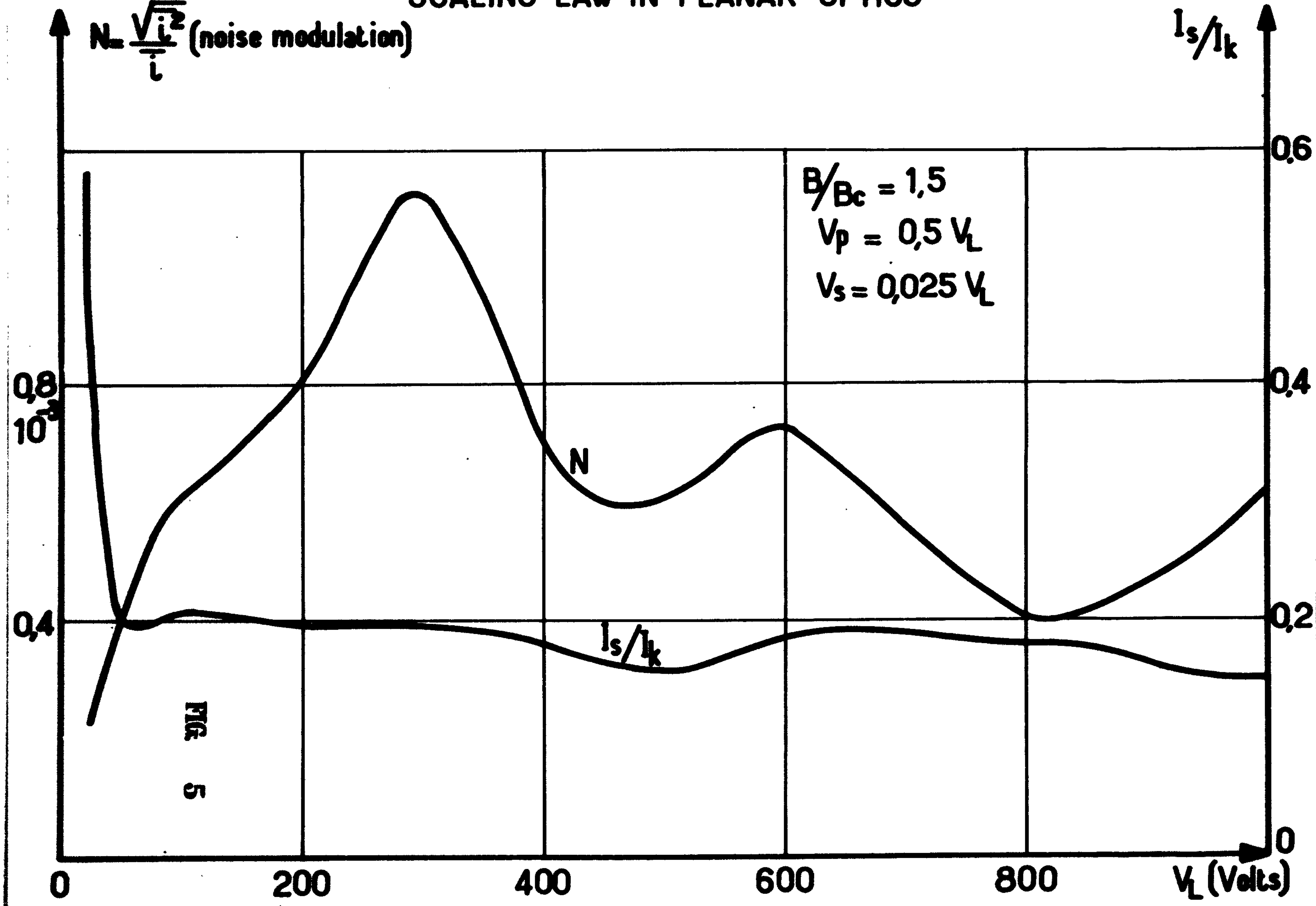


FIG. 5

Correlation coefficient between two collectors { a) near the gun
b) at 10 cm from the gun }

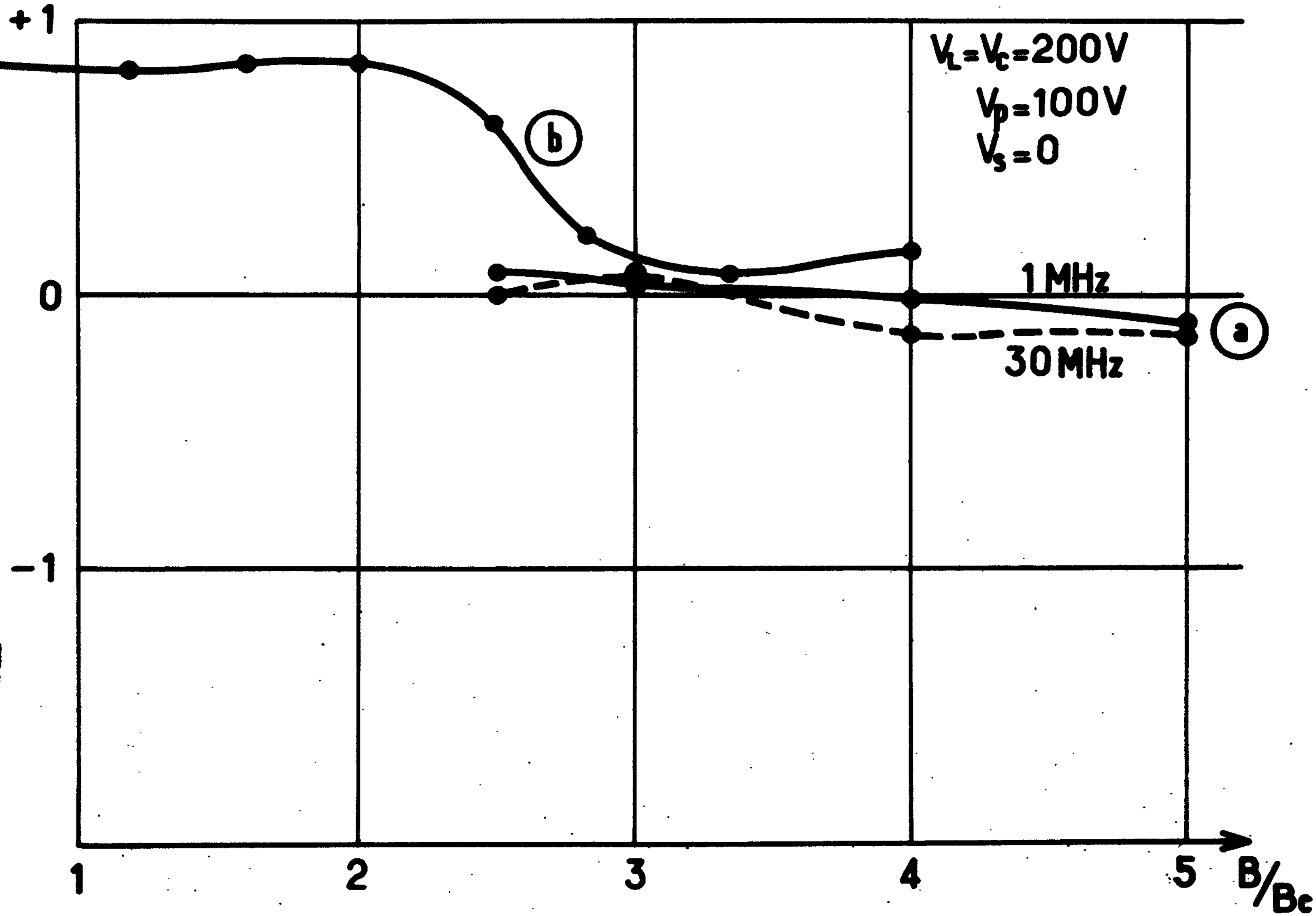
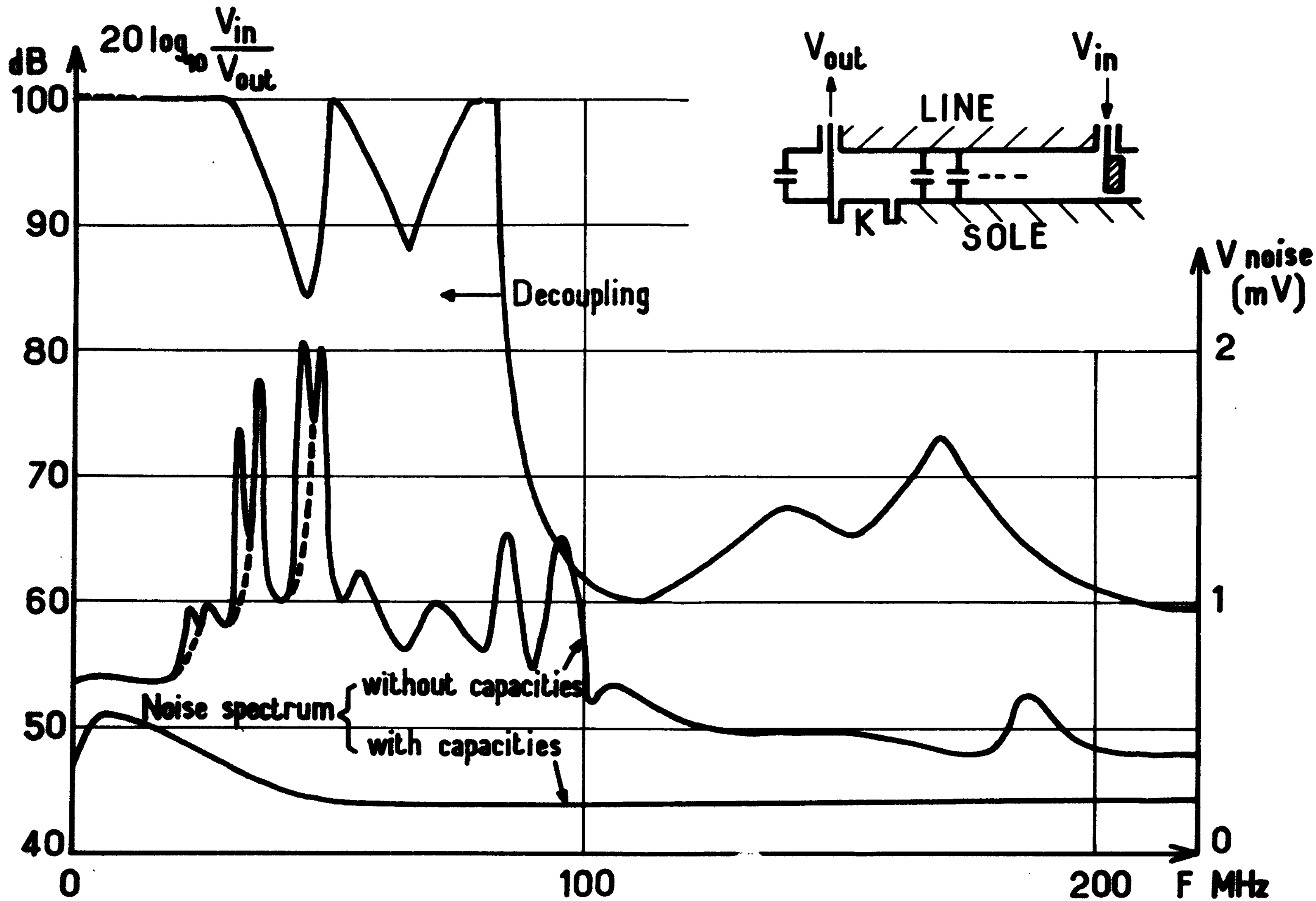


FIG. 6



$20 \log_{10} \frac{V_{in}}{V_{out}}$
dB
100
90
80
70
60
50
40
0
100
200
F MHz
Decoupling
Noise spectrum
without capacities
with capacities
V noise (mV)
2
1
0
V_{out}
V_{in}
LINE
K
L
SOLE

FIG. 7

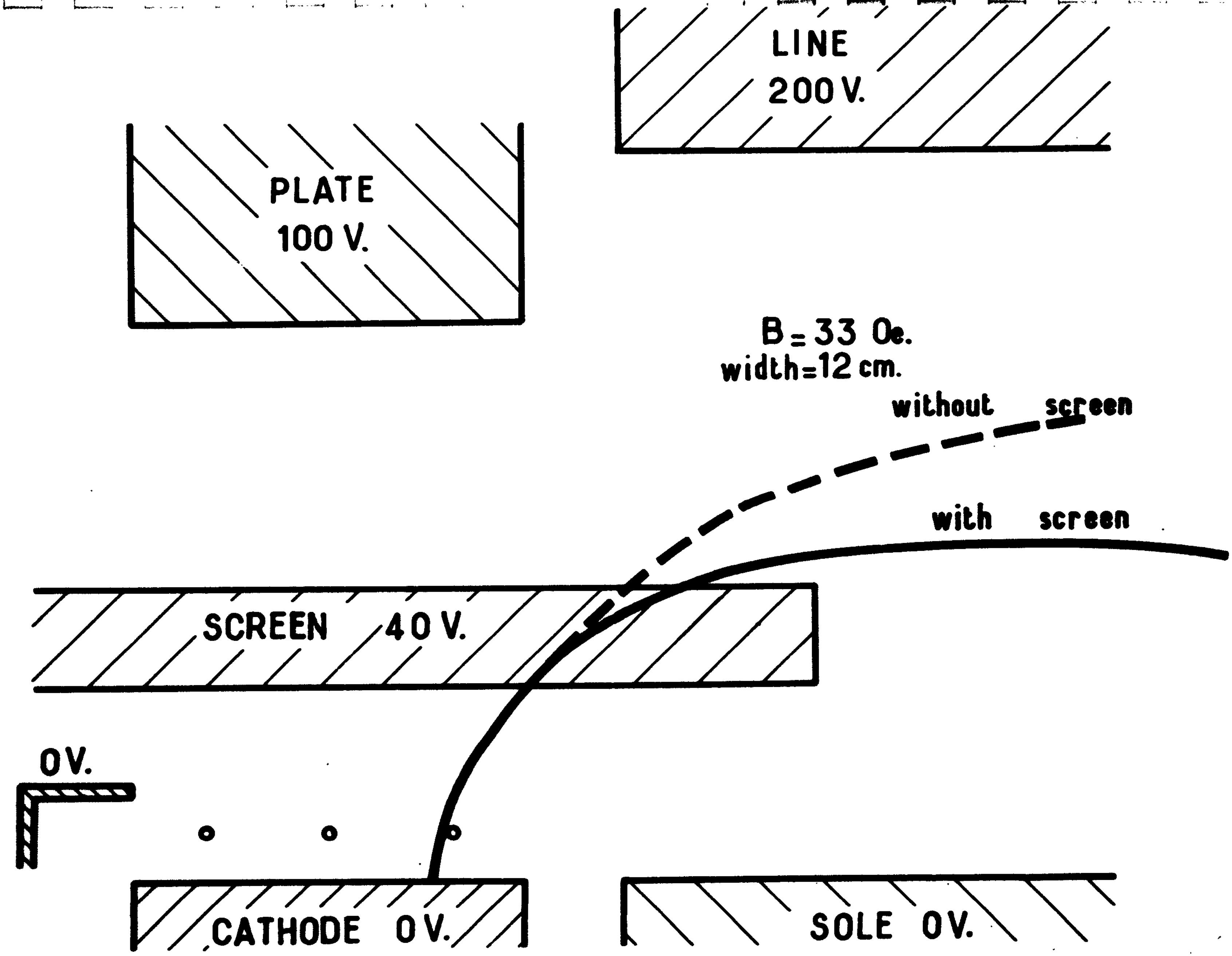

LINE
200 V.
PLATE
100 V.
B = 33 Oe.
width=12 cm.
without screen
with screen
SCREEN 40 V.
0 V.
CATHODE 0 V.
SOLE 0 V.

FIG.

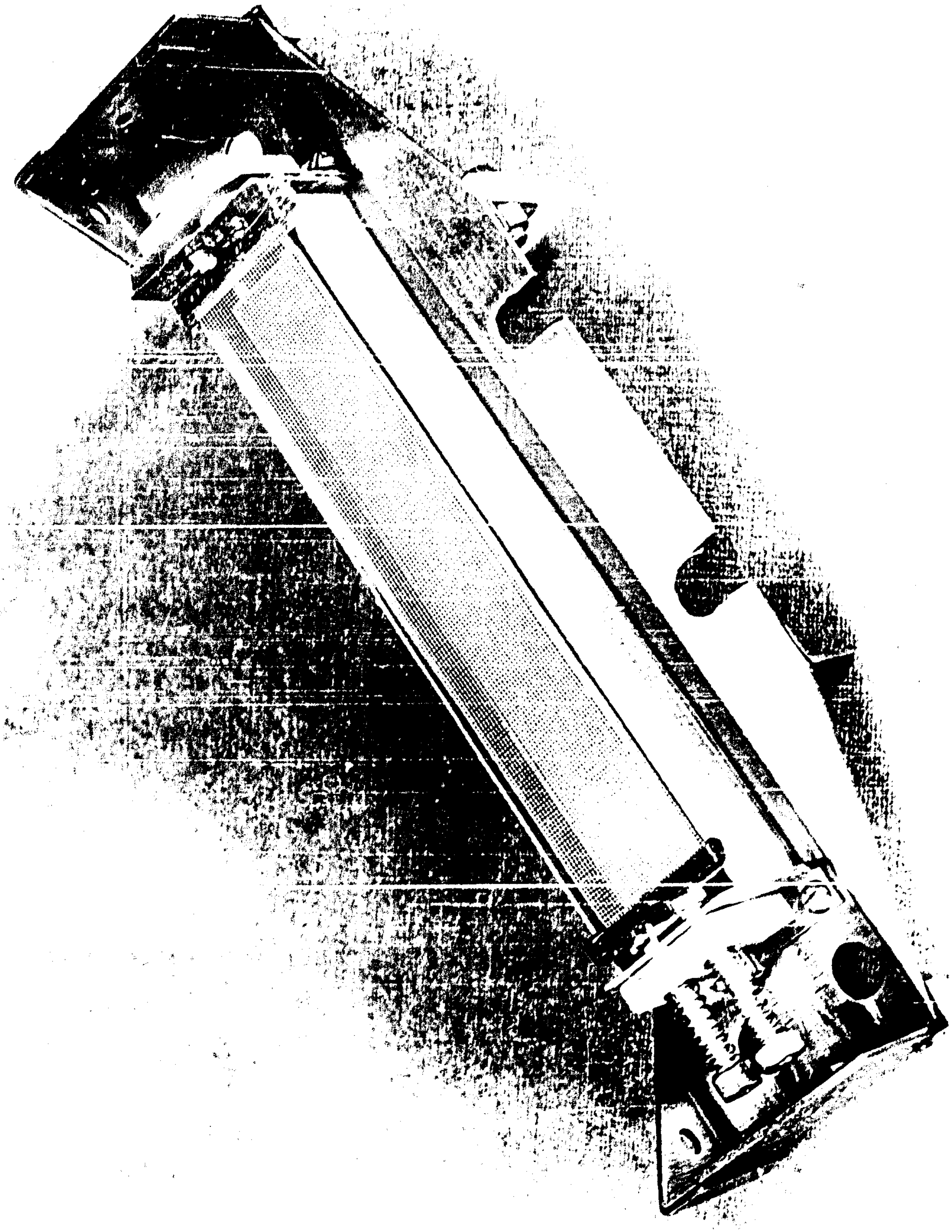


FIG. 9

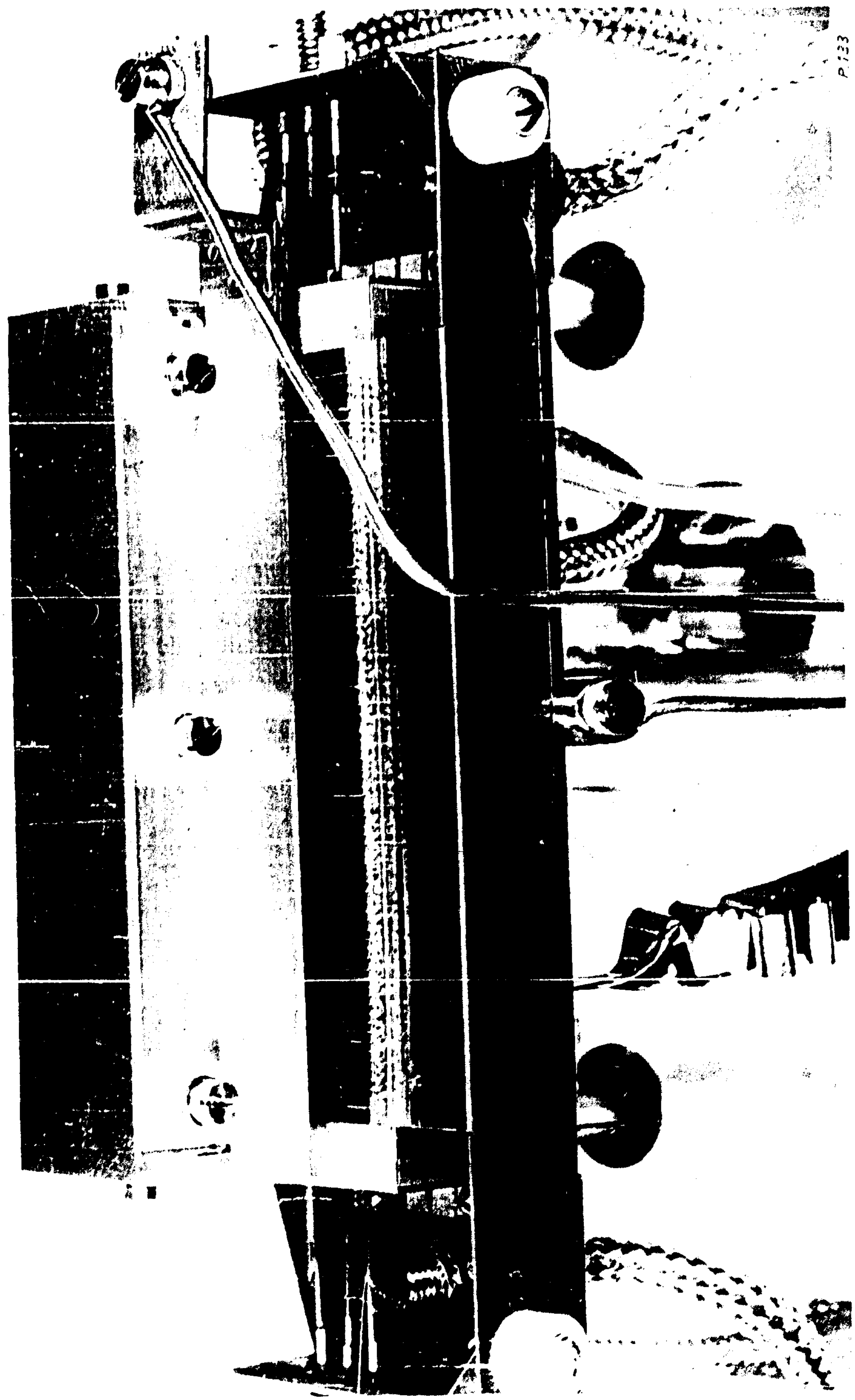


FIG. 3 C.

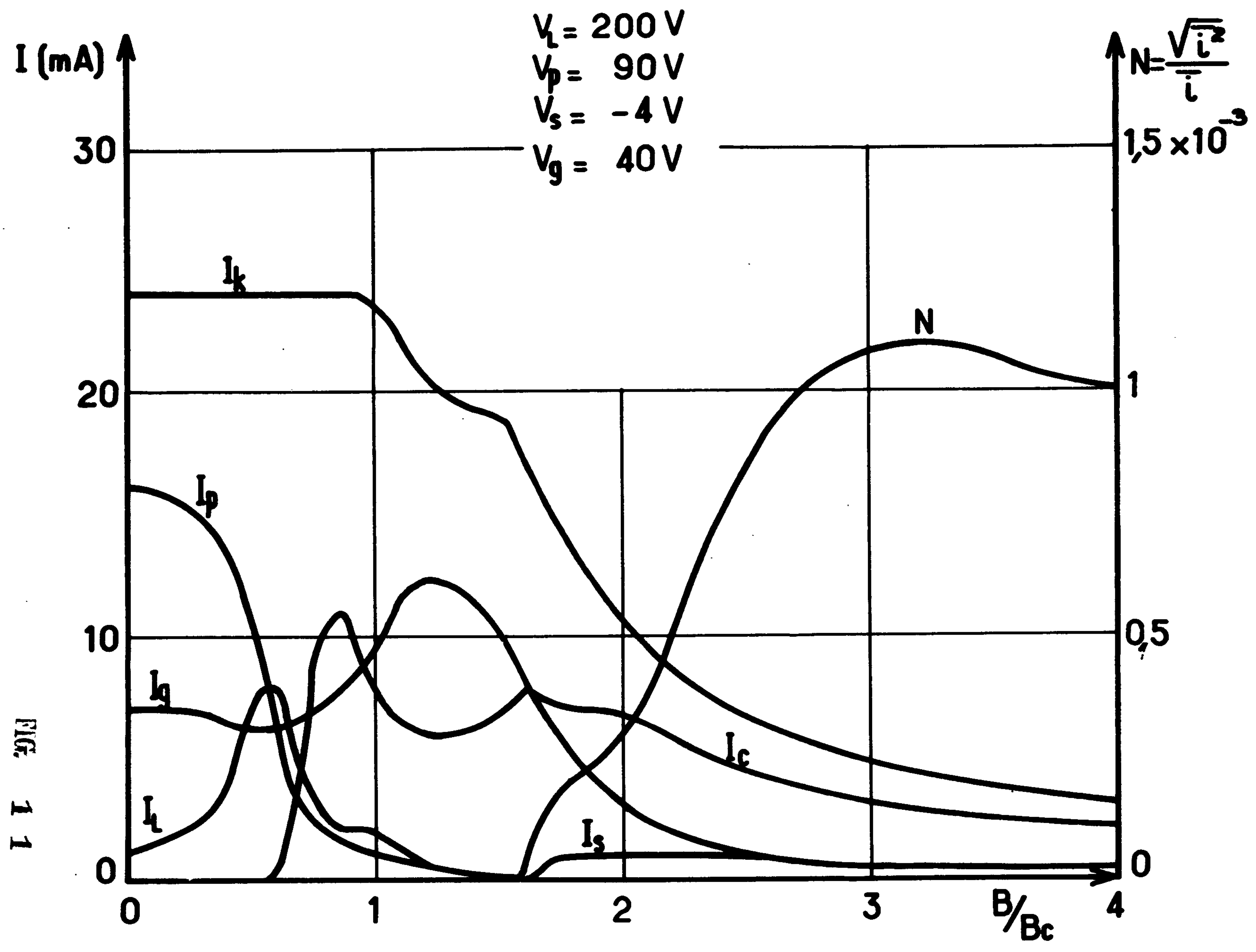

I (mA)
30
20
10
0
V_L = 200 V
V_p = 90 V
V_s = -4 V
V_g = 40 V
$N = \frac{\sqrt{\overline{i^2}}}{\bar{i}}$
1,5 x 10^-3
1
0,5
0
I_k
I_p
I_g
I_t
I_c
I_s
N
0
1
2
3
4
B/Bc

FIG. 1 1

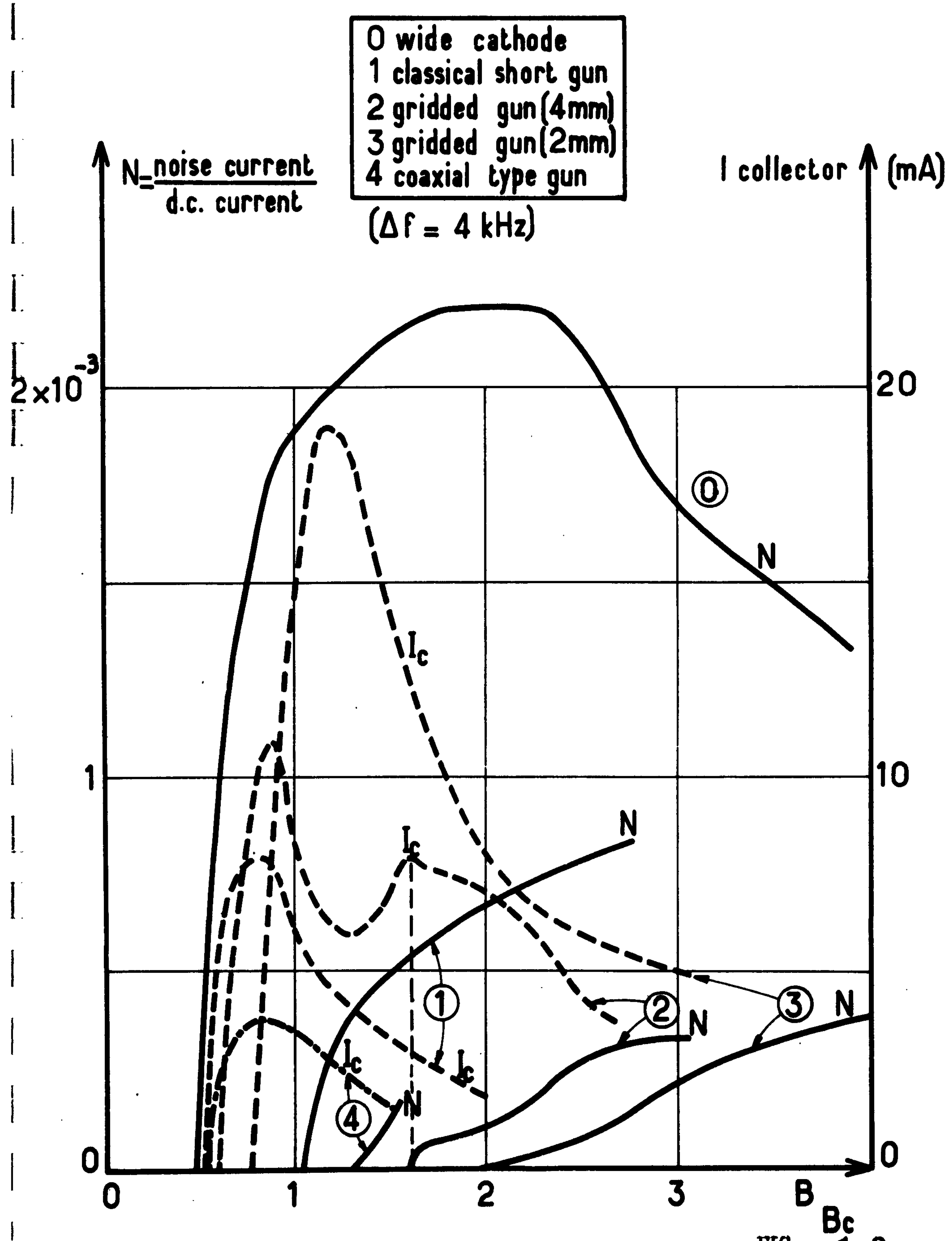

0 wide cathode
1 classical short gun
2 gridded gun (4mm)
3 gridded gun (2mm)
4 coaxial type gun
N = noise current / d.c. current
(Δf = 4 kHz)
I collector (mA)
2×10⁻³
1
0
20
10
0
0
1
2
3
B / Bc
N
I_c
I_c
I_c
I_c
N
N
N
N
0
1
2
3
4

FIG. 1 2

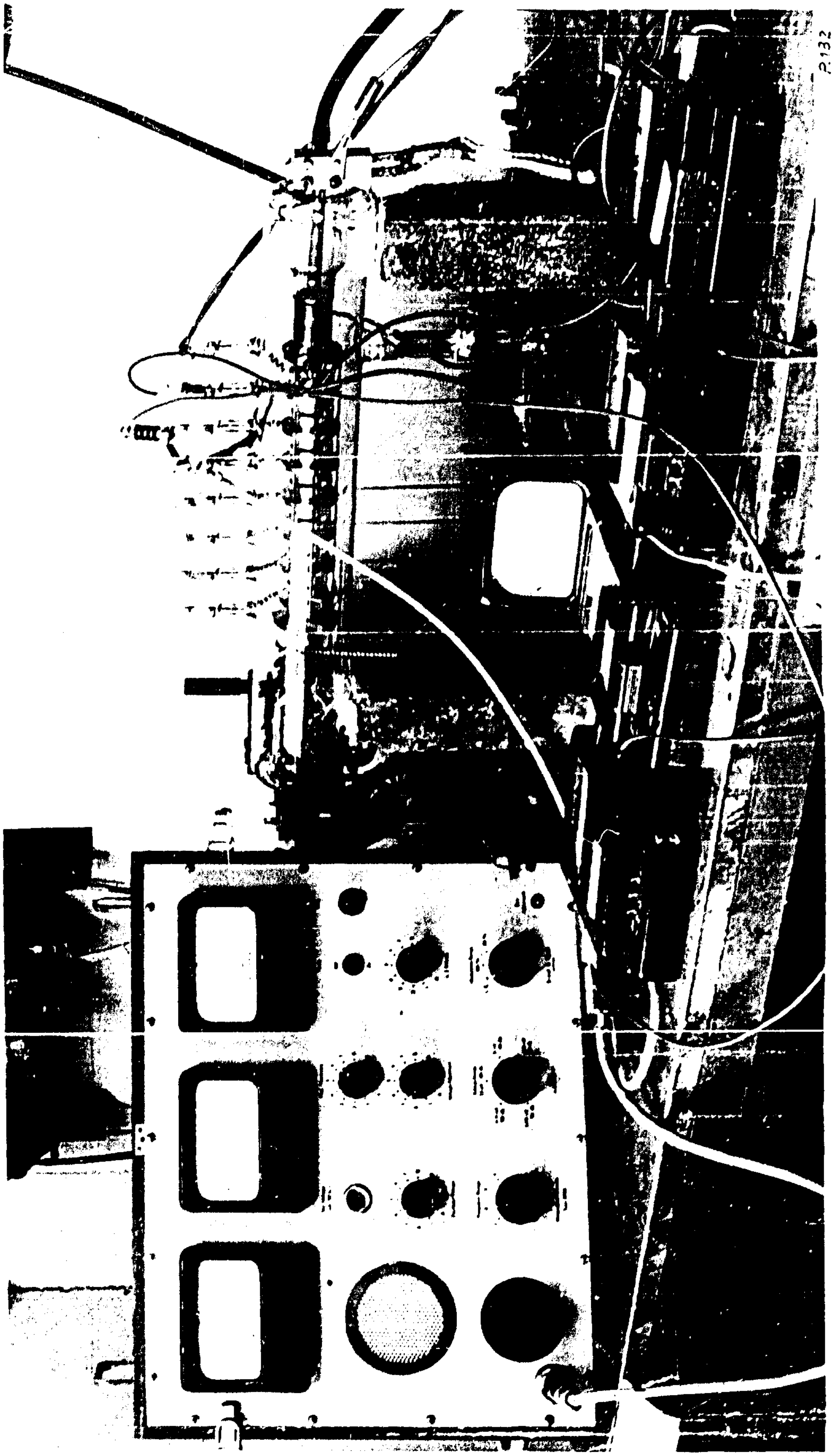


FIG. 1 3

COAXIAL TYPE GUN

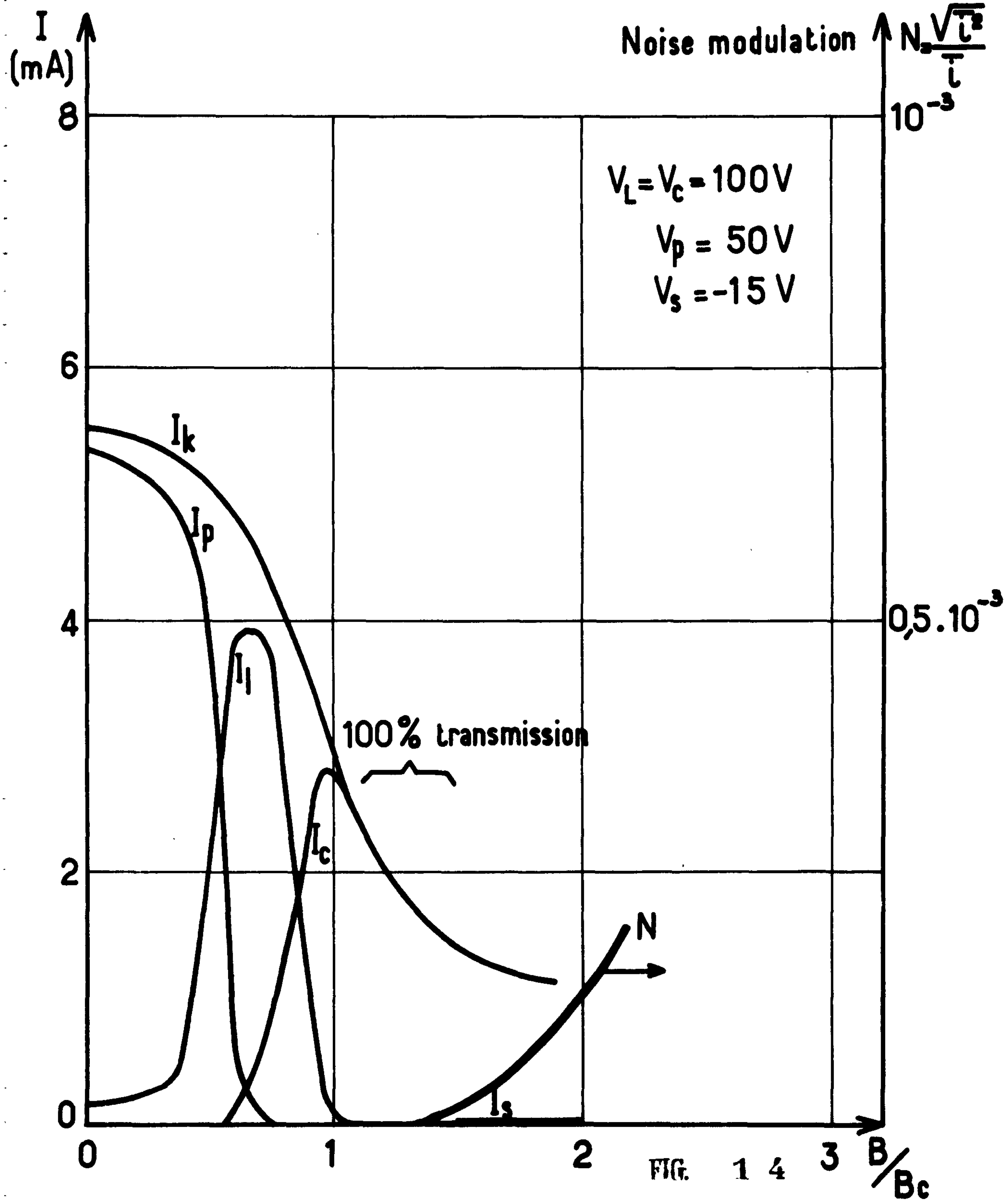
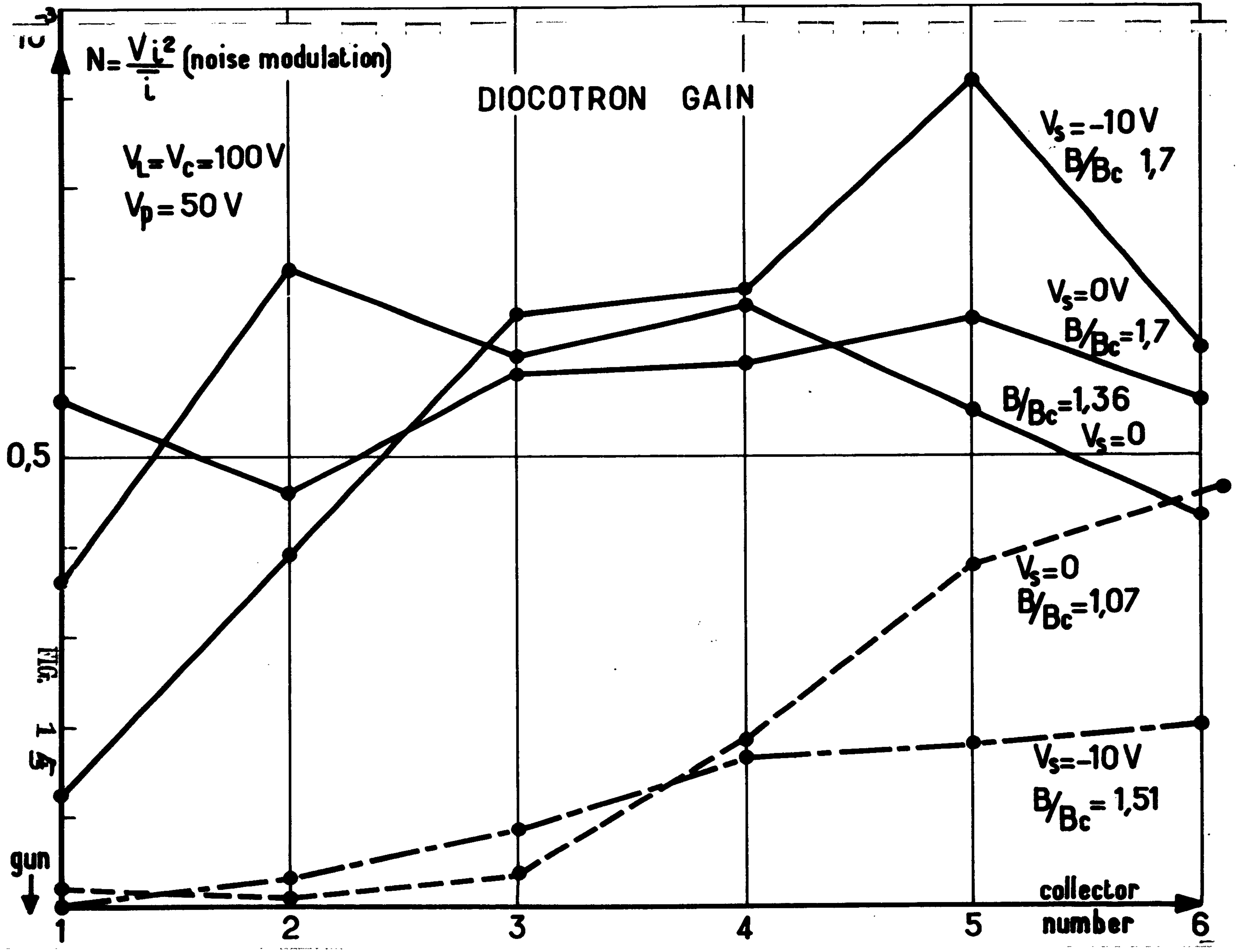


FIG. 14



10^{-3}
$N = \frac{V i^2}{i}$ (noise modulation)
DIOCOTRON GAIN
$V_L = V_c = 100\,V$
$V_p = 50\,V$
$V_s = -10\,V$
B/B_c 1,7
$V_s = 0\,V$
$B/B_c = 1{,}7$
$B/B_c = 1{,}36$
$V_s = 0$
0,5
$V_s = 0$
$B/B_c = 1{,}07$
$V_s = -10\,V$
$B/B_c = 1{,}51$
gun
collector number
1
2
3
4
5
6

FIG. 15

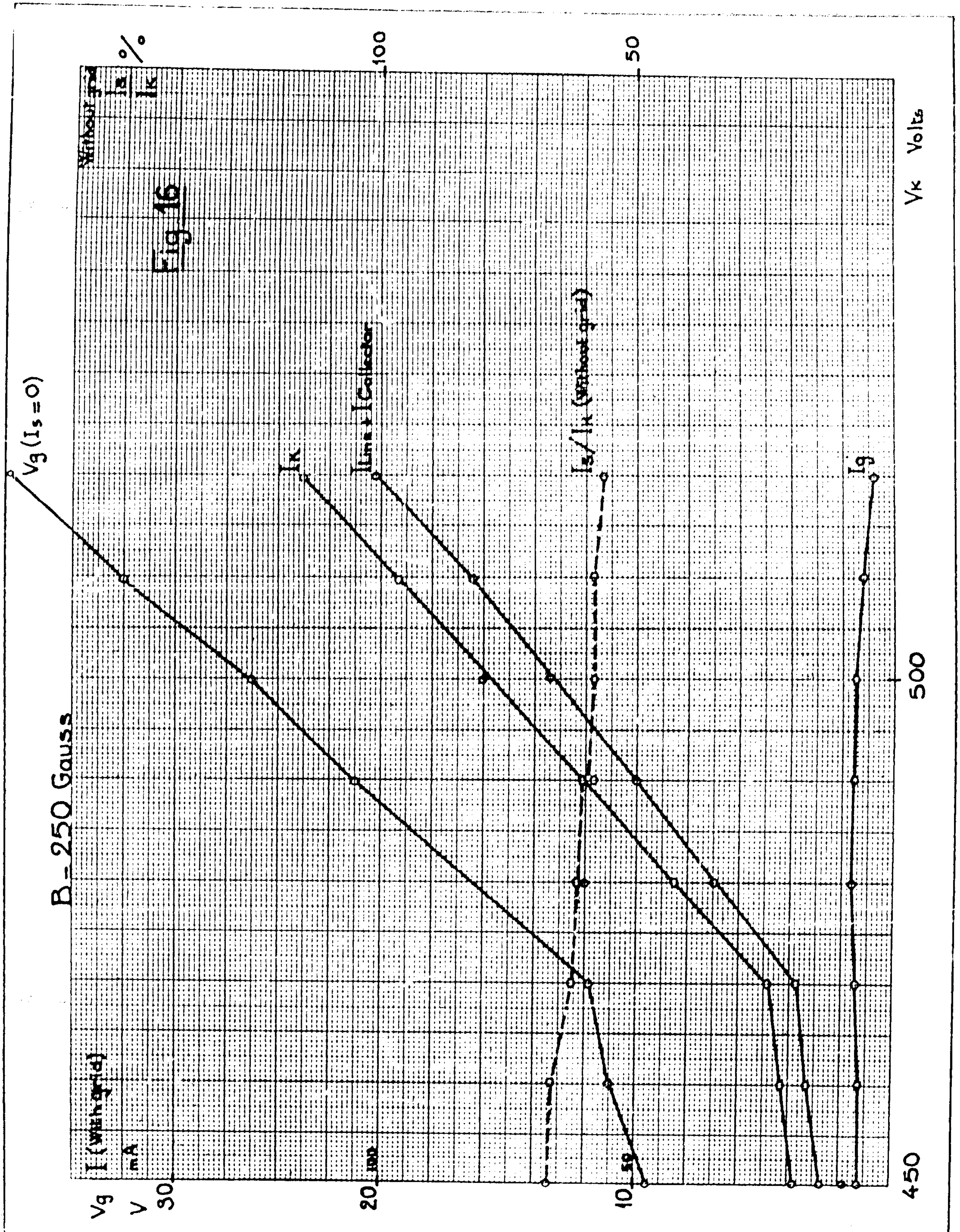
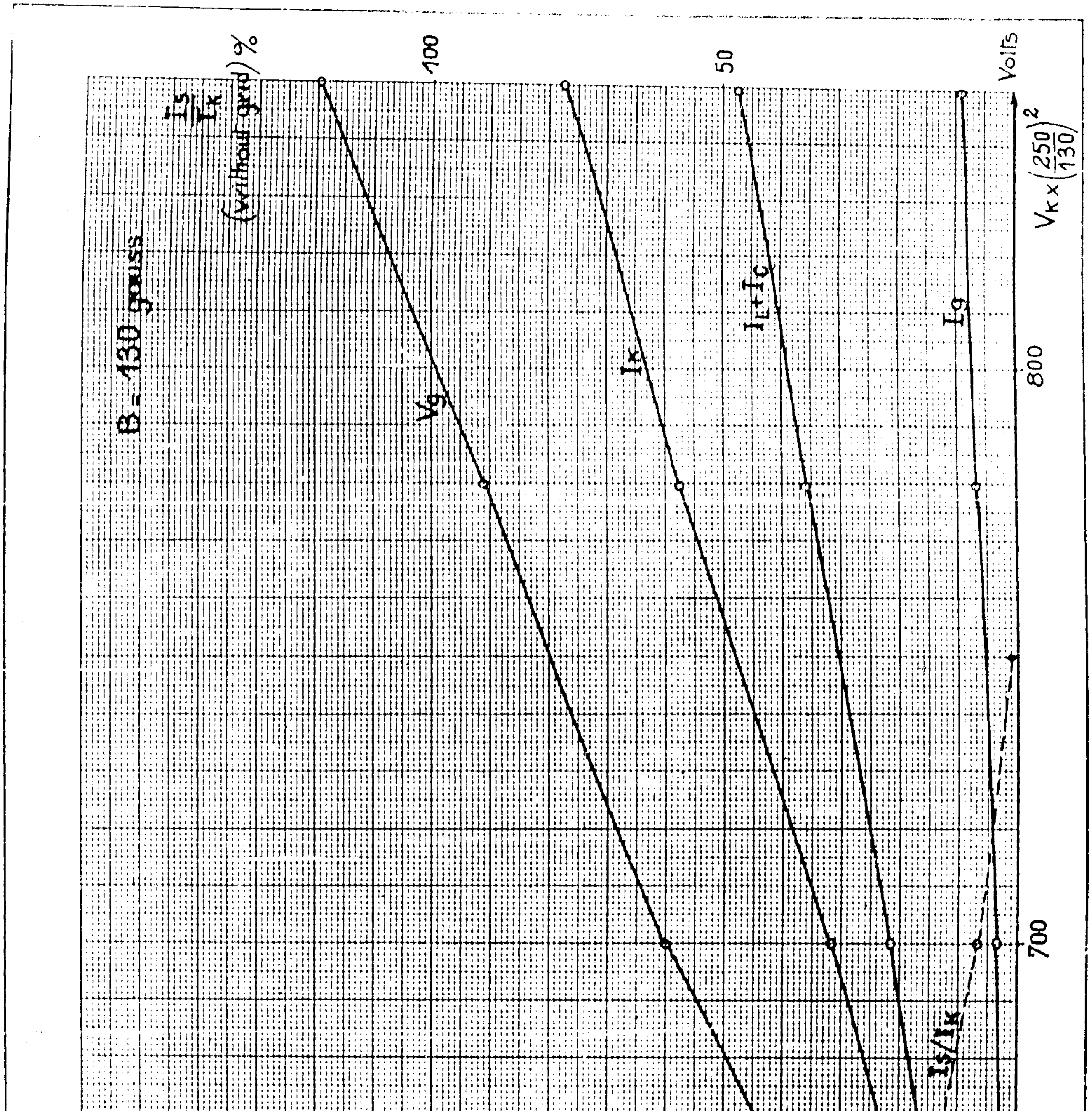

B = 250 Gauss
Vg (Is = 0)
IK
Iline + I Collector
Is/IK (Without grid)
Ig
Fig 16
Without grid
Is/IK %
100
50
I (With grid)
mA
100
50
Vg
V
30
20
10
450
500
VK Volts

Fig. 16



B = 130 gauss
$\frac{I_s}{I_k}$ (without grid) %
100
50
Volts
$V_k \times \left(\frac{250}{130}\right)^2$
700
800
V_g
I_k
$I_L + I_C$
I_g
I_s/I_k
$I \times \left(\frac{250}{130}\right)^2$
mA
500
100
Volts
$V_g \times \left(\frac{250}{130}\right)^2$

Figure 17

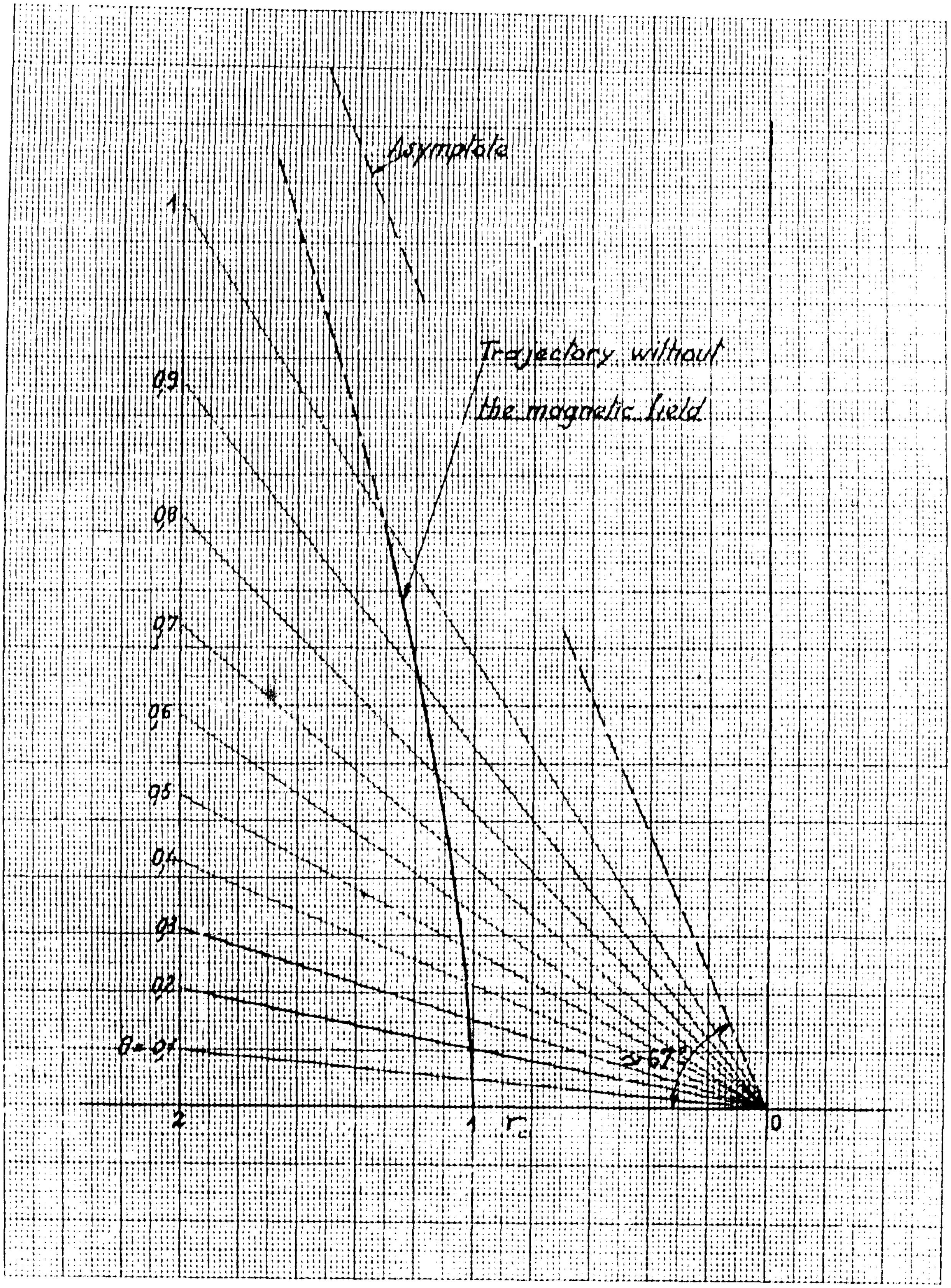
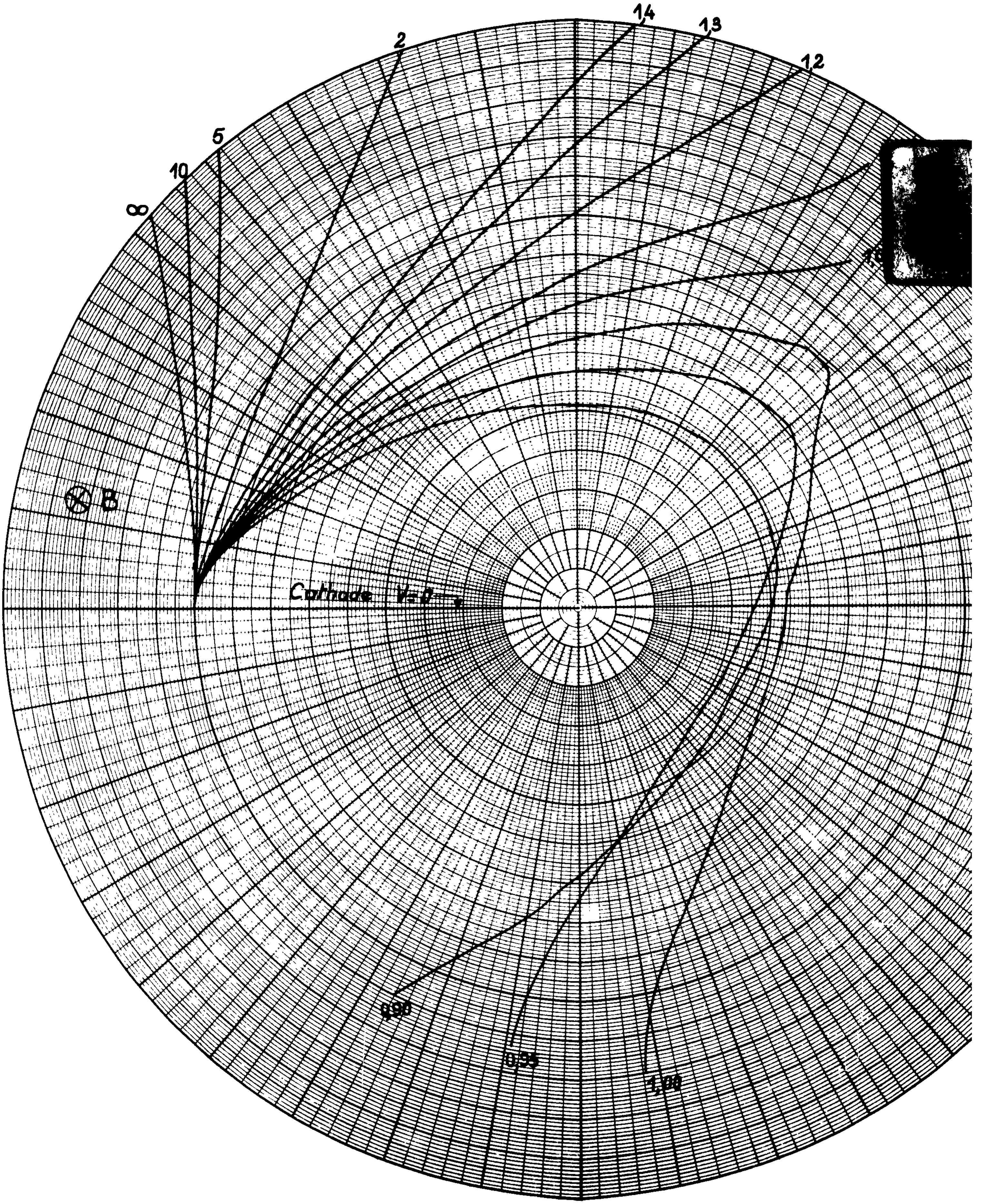

Asymptote
Trajectory without
the magnetic field
1
0,9
0,8
0,7
0,6
0,5
0,4
0,3
0,2
θ = 0,1
≈ 67°
2
1
r_c
0

Figure 18



2
5
10
∞
14
13
12
⊗B
Cathode V=0
0,90
0,95
1,00

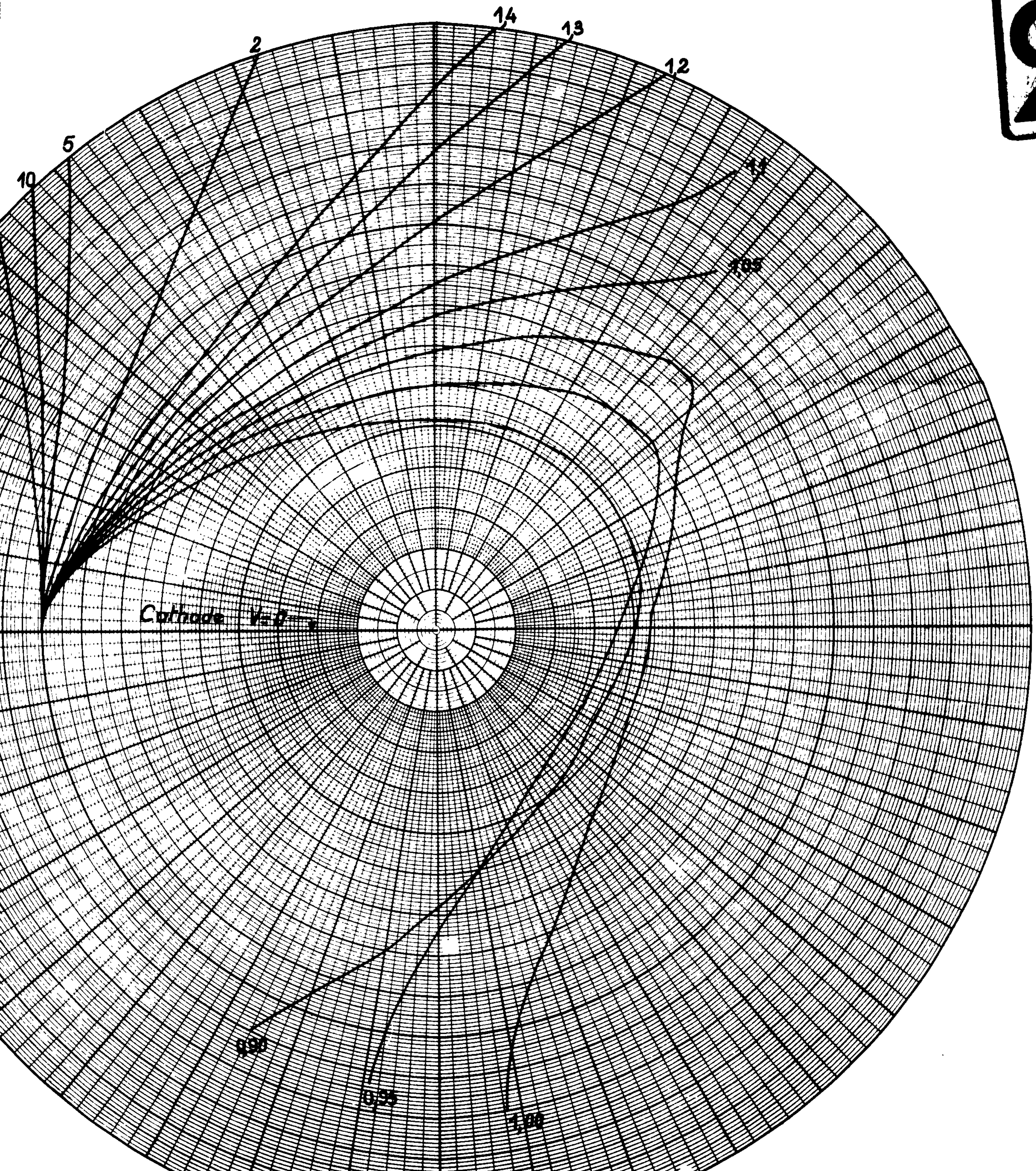
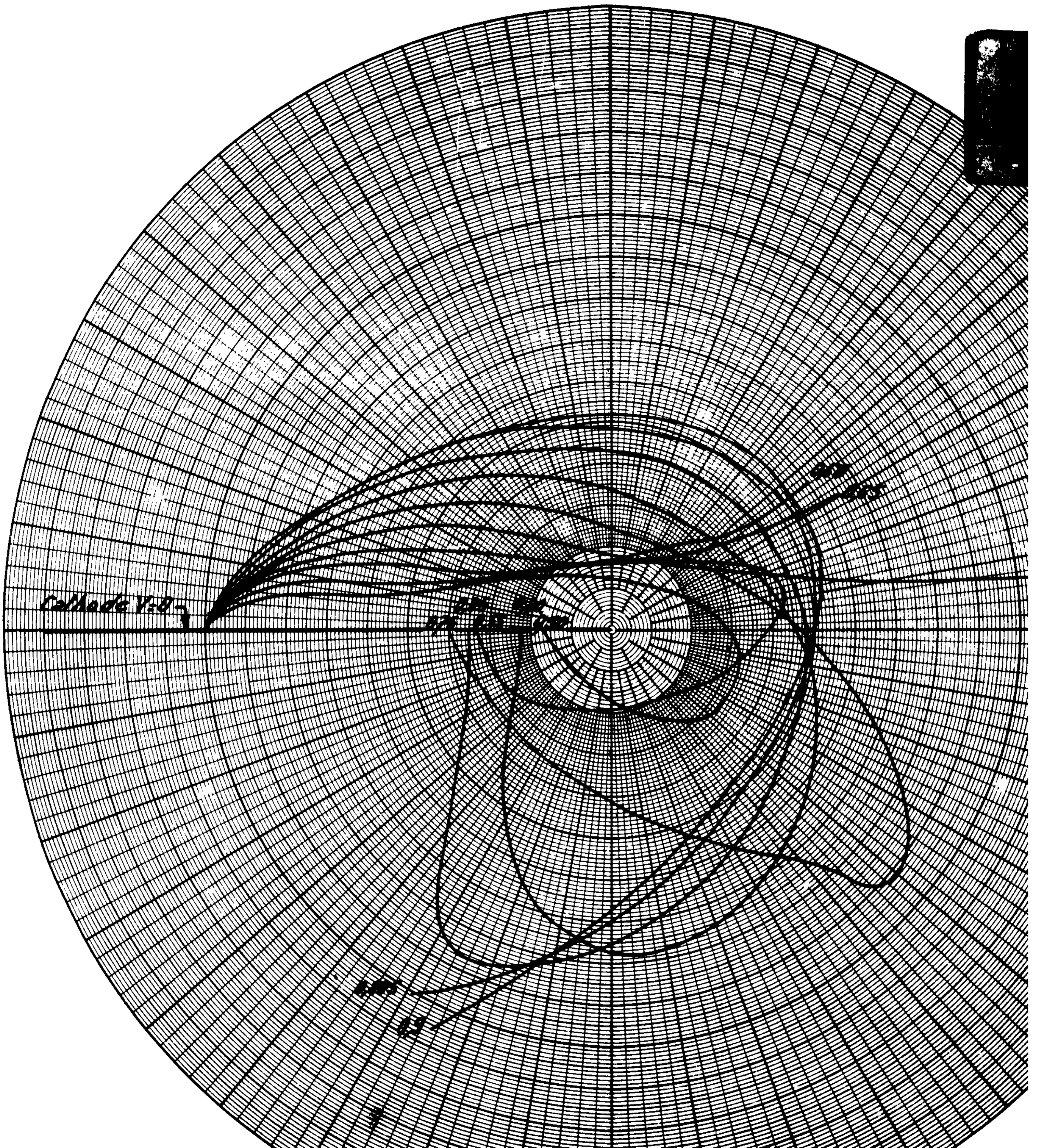


Figure 19


Cathode V=0

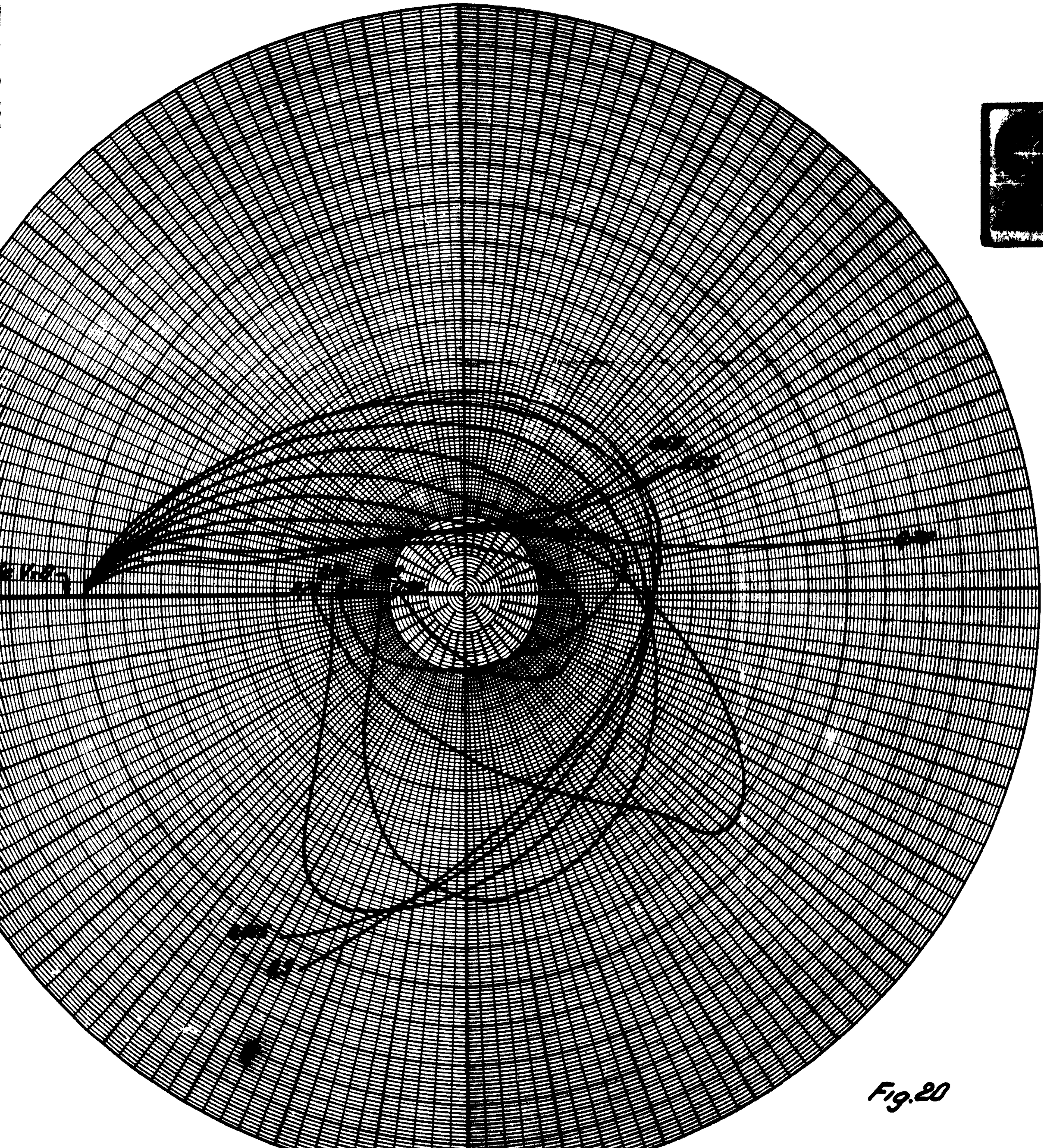
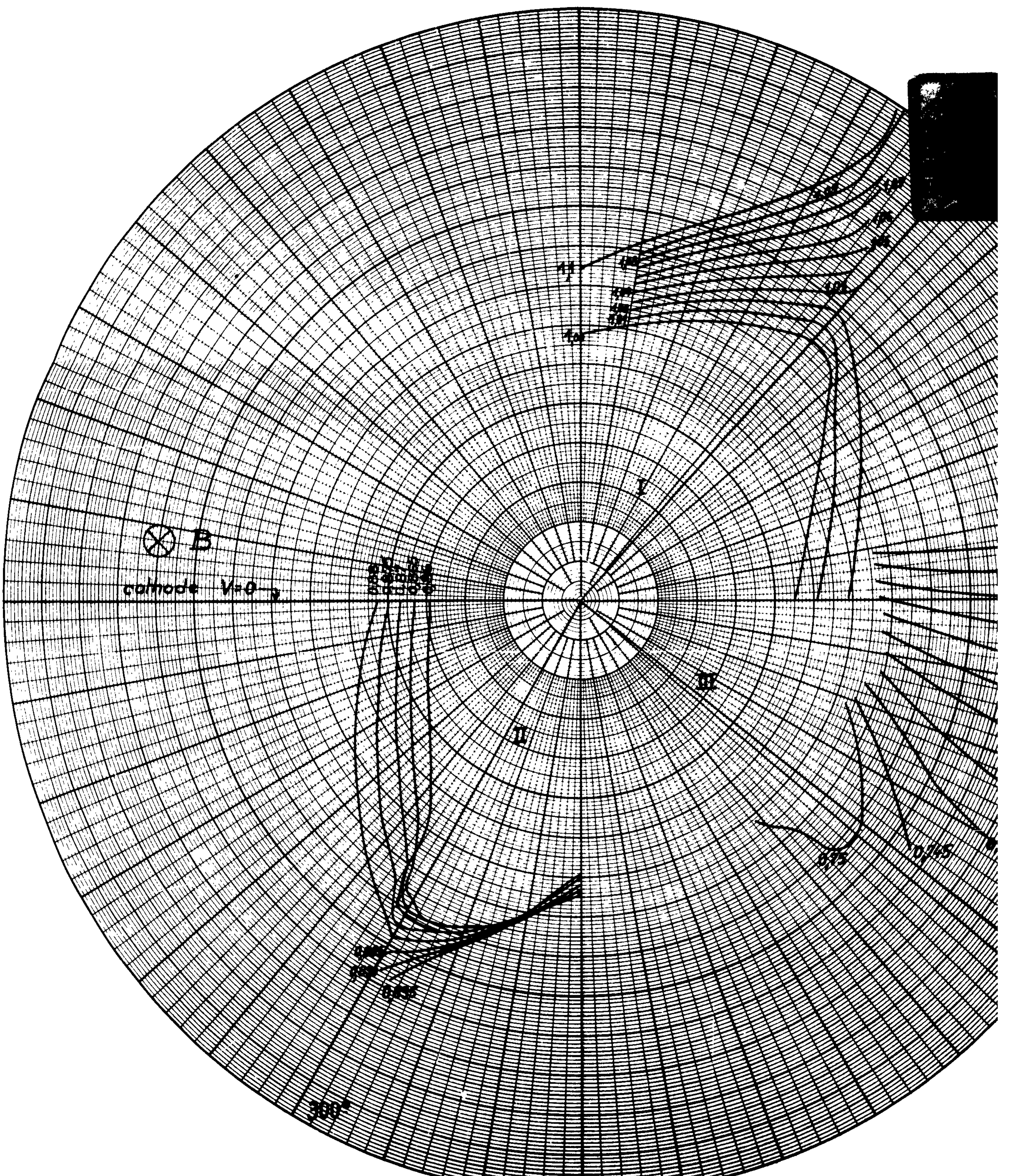


Fig. 20



B
cathode V=0
I
II
III
0,75
0,765
300°

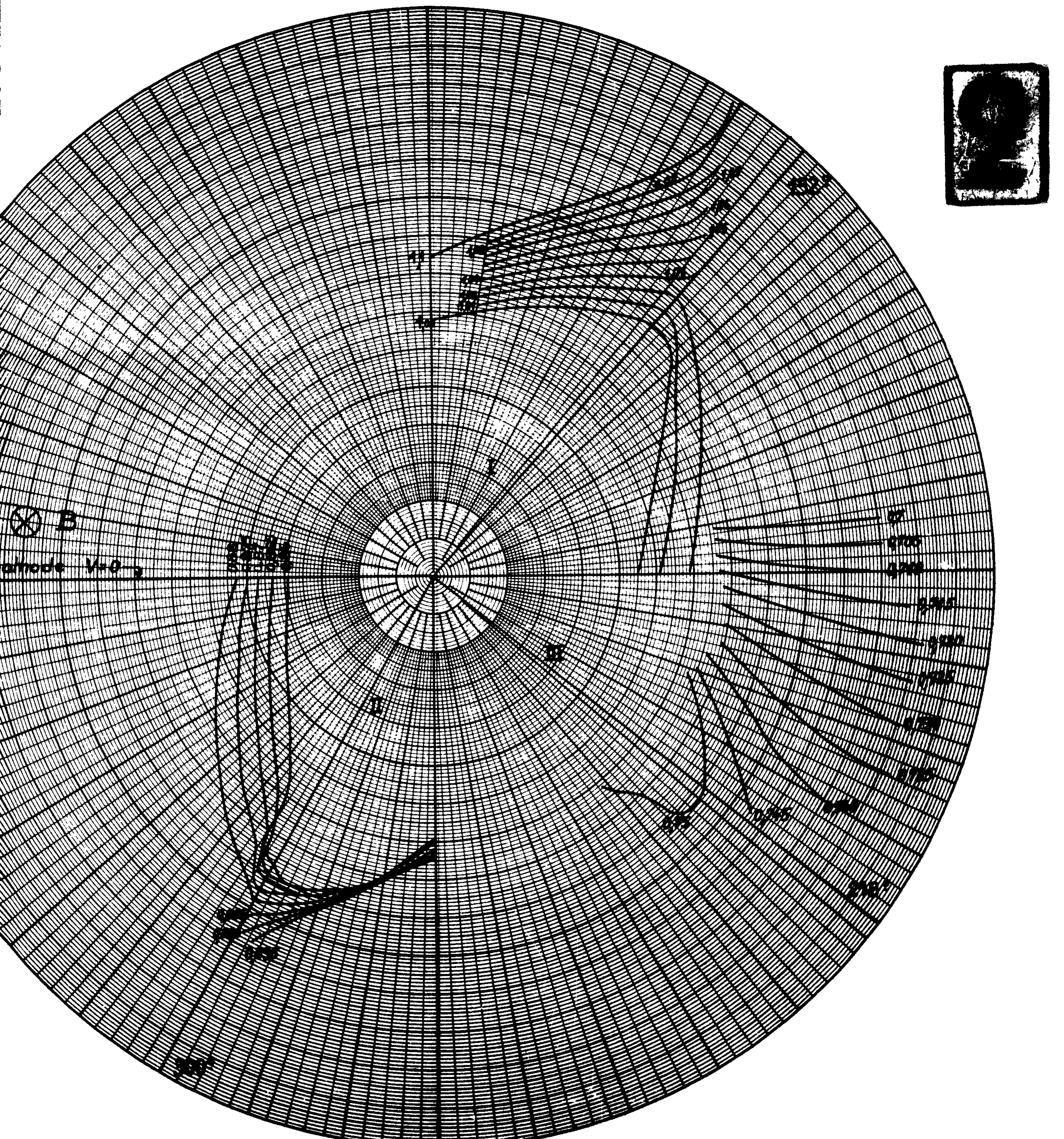
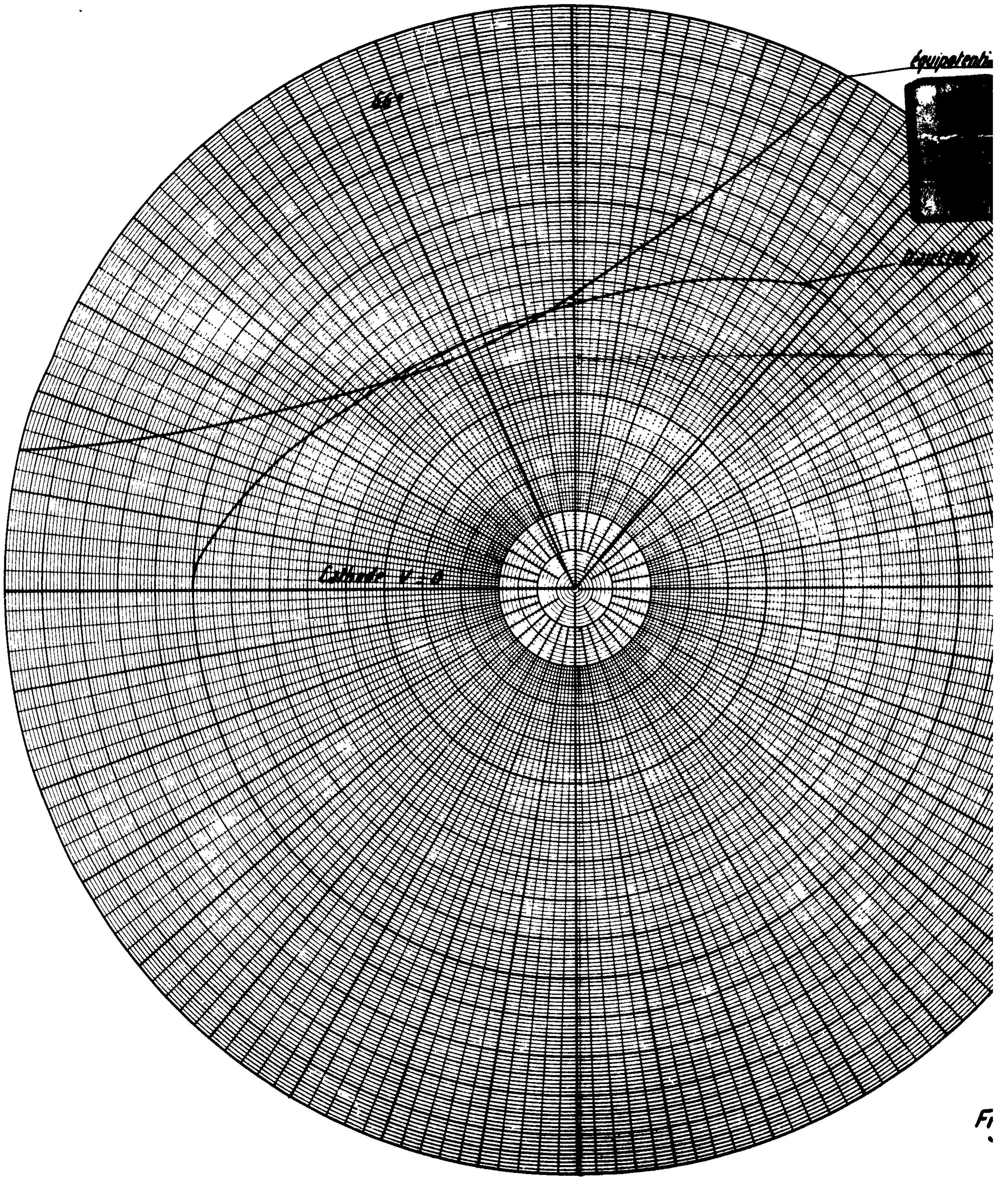


Figure 21



Fi

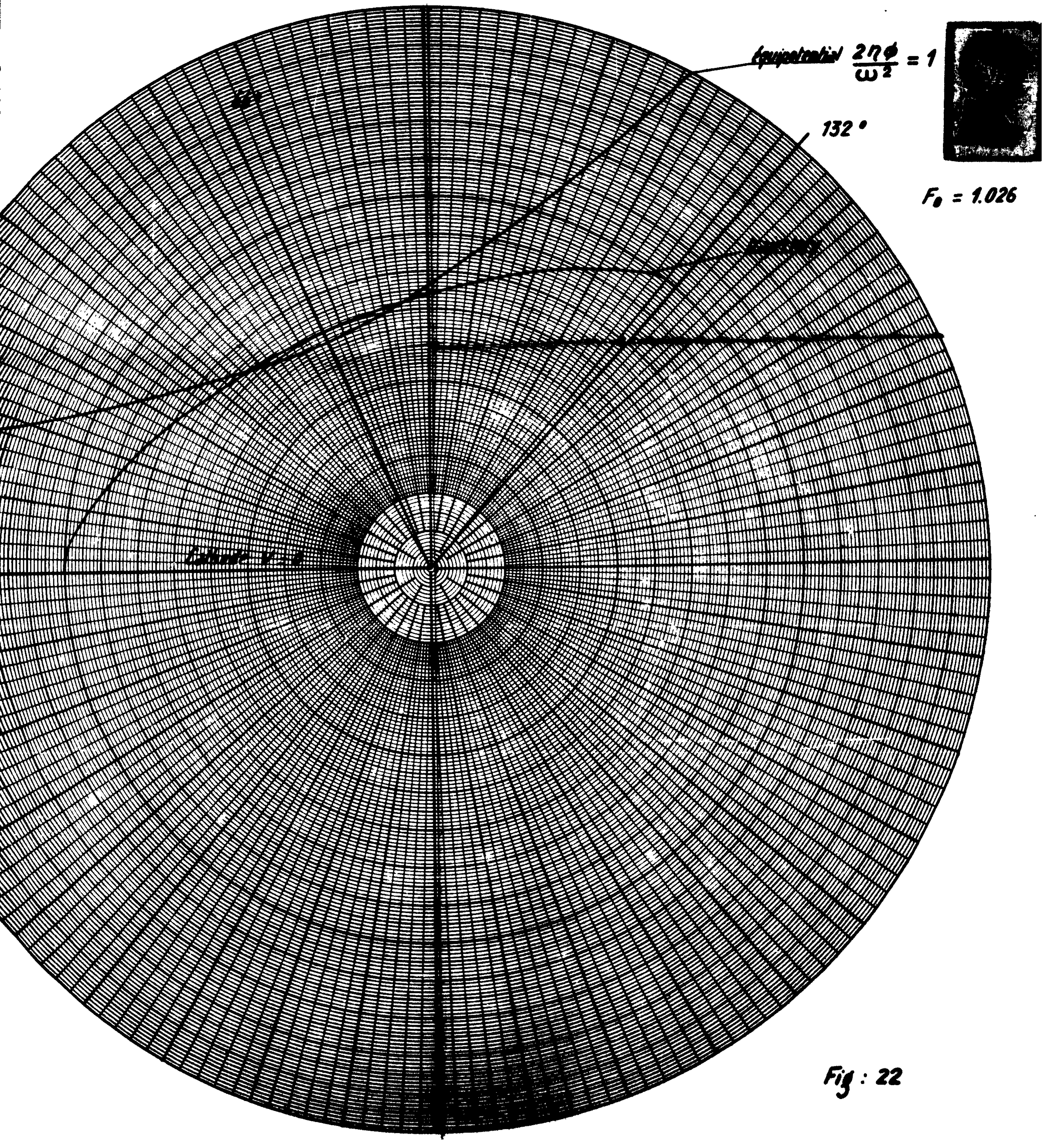
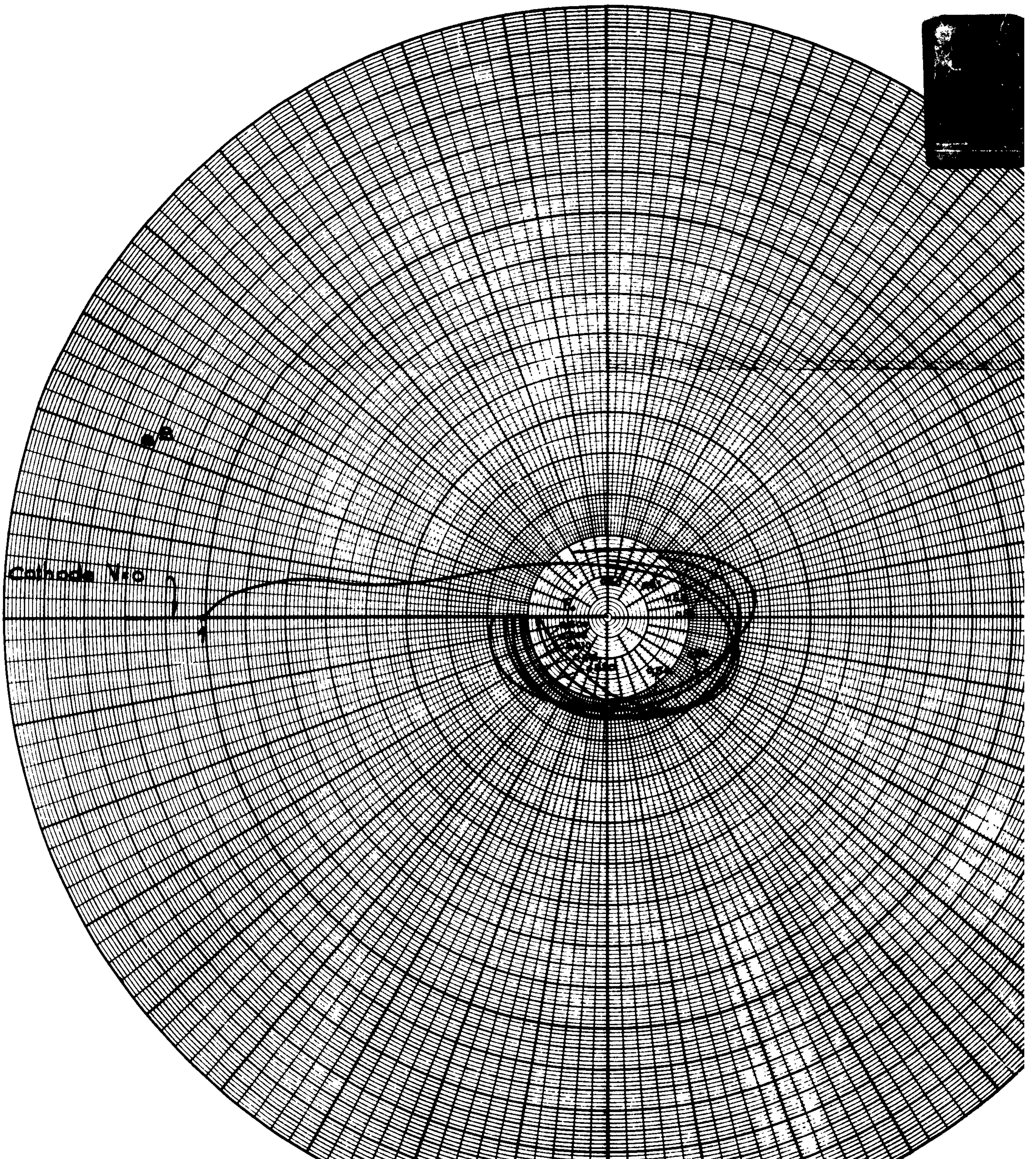


Fig : 22



Fi

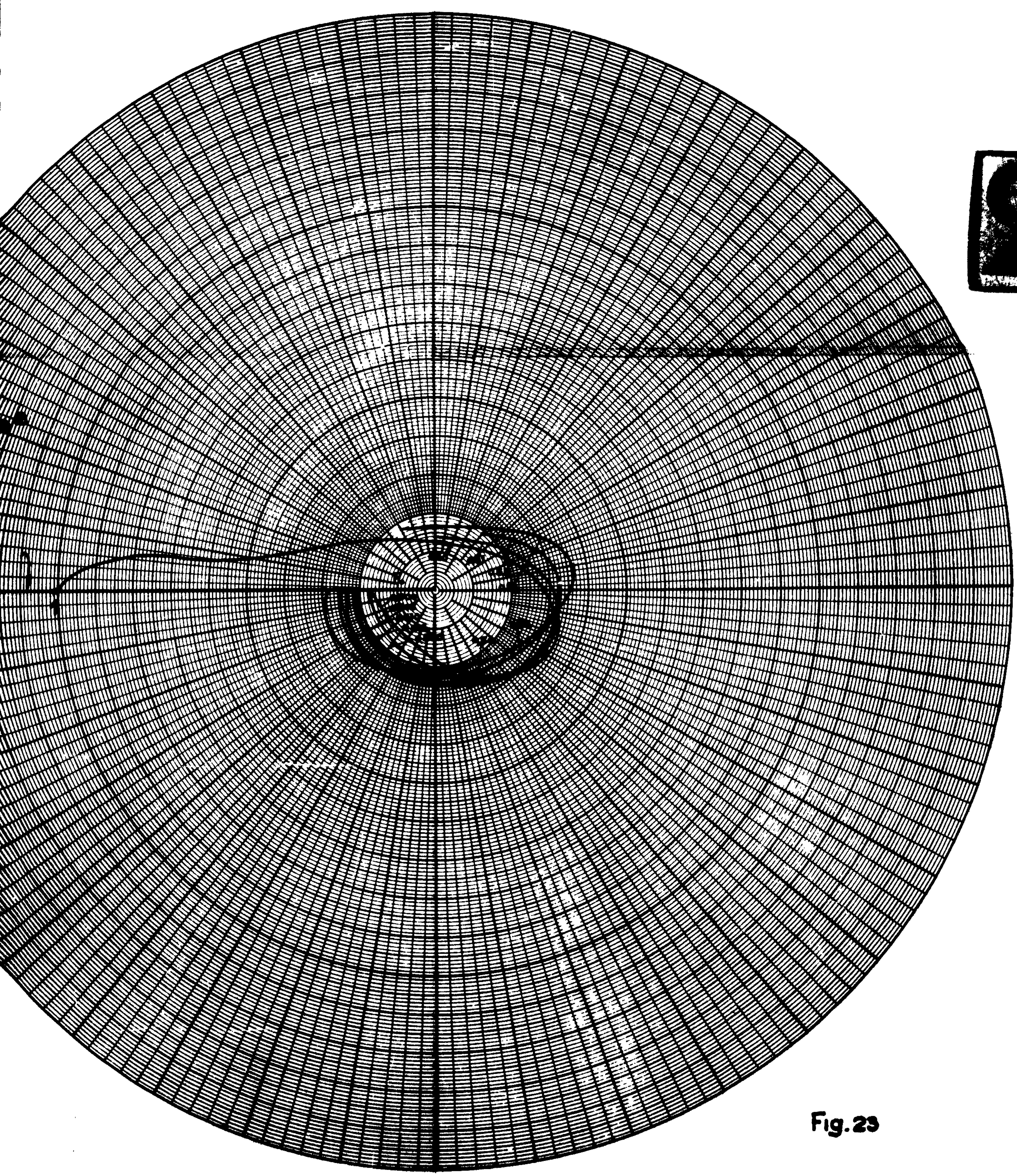


Fig. 23

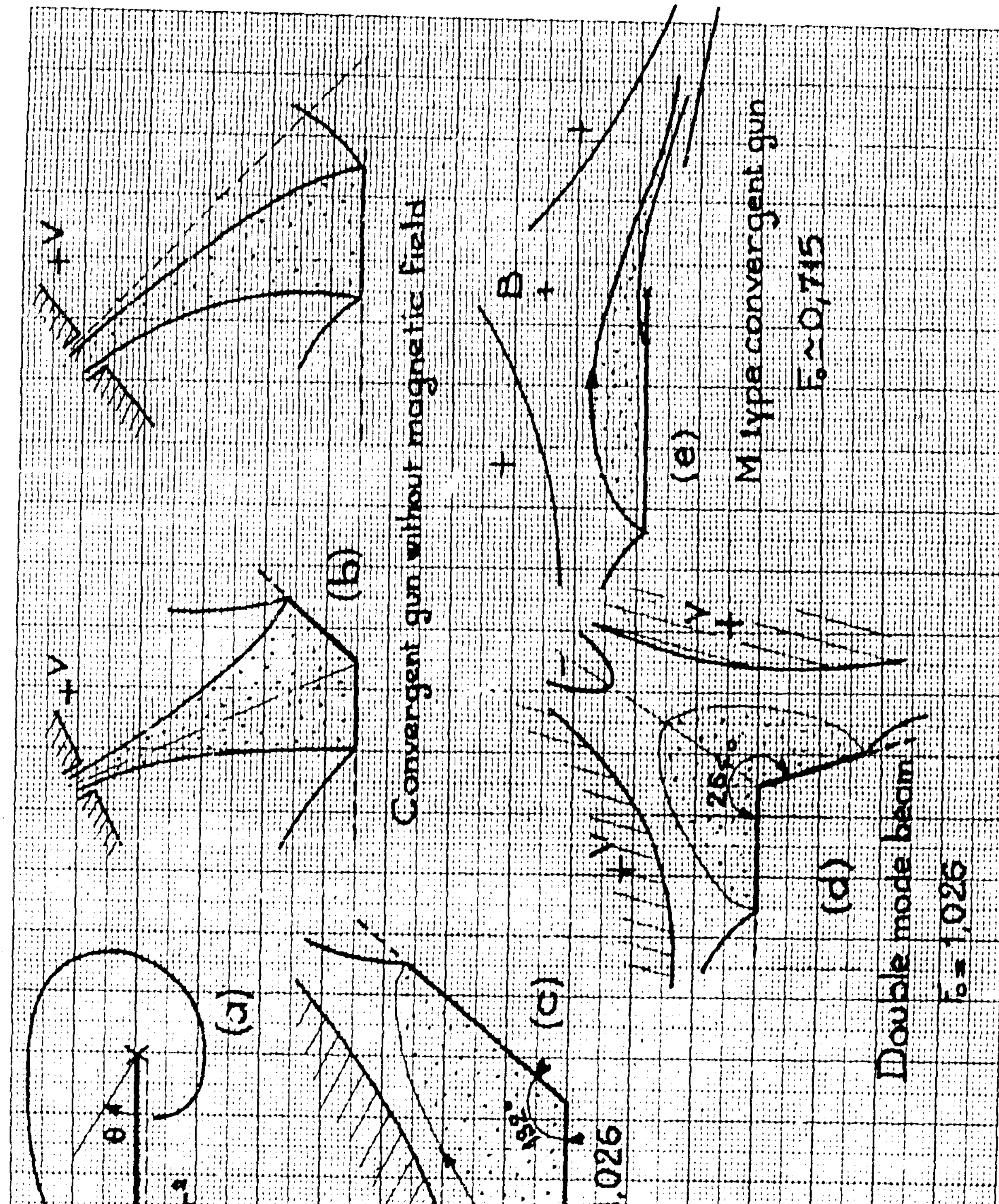

Convergent gun without magnetic field
(a)
(b)
(c)
(d)
(e)
V=0
V=Kr²
θ
+V
18°
F₀ = 1,026
26,5°
Double mode beam
F₀ = 1,026
B
M type convergent gun
F₀ ~ 0,715

Fig. 24